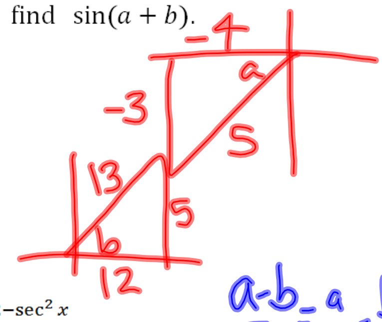
1. Given that $\sin a = -\frac{3}{5}$, $a \in QIII$, and $\tan b = \frac{5}{12}$, $b \in QI$, find $\sin(a+b)$.

$\sin(a+b) = \sin a \cos b + \cos a \sin b$

$= \left(\frac{-3}{5}\right)\left(\frac{12}{13}\right) + \left(\frac{-4}{5}\right)\left(\frac{5}{13}\right)$

$= \frac{-36}{65} + \frac{-20}{65} = \boxed{\frac{-56}{65}}$



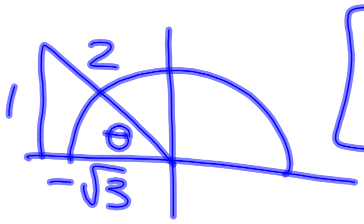
2. Simplify and express as a single trigonometric function $\frac{2-\sec^2 x}{\sec^2 x}$.

$\frac{a-b}{c} = \frac{a}{c} - \frac{b}{c}$

$\frac{2}{\sec^2 x} - \frac{\sec^2 x}{\sec^2 x} = \frac{2}{\frac{1}{\cos^2 x}} - 1 = 2 \cdot \frac{\cos^2 x}{1} - 1 = \boxed{\cos 2x}$

$\frac{2 - \frac{1}{\cos^2 x}}{\frac{1}{\cos^2 x}} = \left(2 - \frac{1}{\cos^2 x}\right)\left(\frac{\cos^2 x}{1}\right) = 2\cos^2 x - 1 =$

3. Evaluate $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$. Give the answer in radians.



$\boxed{\frac{5\pi}{6}}$

Solve for $x \in [0, 2\pi)$. $(a-b)^2 = a^2 - 2ab + b^2$

$(\sin x - \cos x)^2 = (1)^2$

$x = 2$

$x^2 = 4$

$(\sin x - \cos x)(\sin x - \cos x) = 1$

$\sin^2 x - \underbrace{2\sin x \cos x}_{\sin 2x} + \cos^2 x = 1$

$\underbrace{(\sin^2 x + \cos^2 x)}_{1} - 2\sin x \cos x = 1$

$1 - \sin 2x = 1$

$\sin(2x) = 0$

*squaring both sides of an equation may introduce extraneous solutions, so make sure to plug answers back into original eq. to check.

$2x = 0, \pi, 2\pi, 3\pi$

$x = \cancel{0}, \boxed{\frac{\pi}{2}}, \boxed{\pi}, \cancel{\frac{3\pi}{2}}$

$0 \le x < 2\pi$

$0 \le 2x < 4\pi$

$\sin 0 - \cos 0 = 0 - 1 = -1$

$\sin\frac{\pi}{2} - \cos\frac{\pi}{2} = 1 - 0 = 1$

$\sin\pi - \cos\pi = 0 - (-1) = 1$

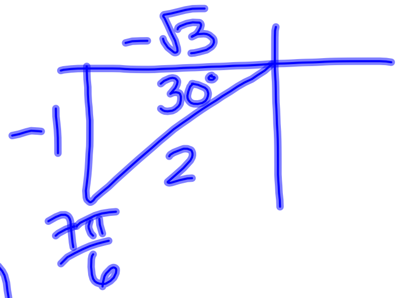
$\sin\frac{3\pi}{2} - \cos\frac{3\pi}{2} = -1 - 0 = -1$

1. Use the half-angle identity to evaluate $\tan\frac{7\pi}{12}$ exactly.

$$\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta}$$

$$\tan\frac{7\pi}{12} = \tan\frac{\frac{7\pi}{12}\cdot 2}{2} = \tan\frac{\left(\frac{7\pi}{6}\right)}{2}$$

$$= \frac{1-\cos\frac{7\pi}{6}}{\sin\frac{7\pi}{6}} = \frac{1-\left(-\frac{\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}\right)}$$



$$=\left(1+\frac{\sqrt{3}}{2}\right)\left(\frac{-2}{1}\right) = \boxed{-2-\sqrt{3}}$$

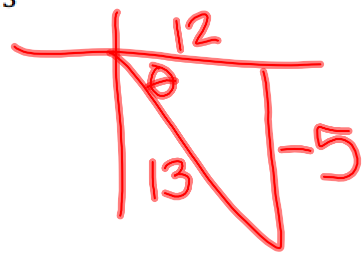
2. Find the exact value of $\cos 212° \cos 122° + \sin 212° \sin 122°$.

$\cos a \cos b + \sin a \sin b = \cos(a-b)$

$$= \cos(212° - 122°) = \cos 90° = \boxed{0}$$

3. Find $\sin 2\theta$, $\cos 2\theta$, *and* $\tan 2\theta$ given that $\cos\theta = \frac{12}{13}$ and θ is in Quadrant IV.

(& the quadrant in which 2θ lies)



$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$= 2\left(\frac{-5}{13}\right)\left(\frac{12}{13}\right) = \boxed{\frac{-120}{169}}$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$= \left(\frac{12}{13}\right)^2 - \left(\frac{-5}{13}\right)^2 = \frac{144}{169} - \frac{25}{169} = \boxed{\frac{119}{169}}$$

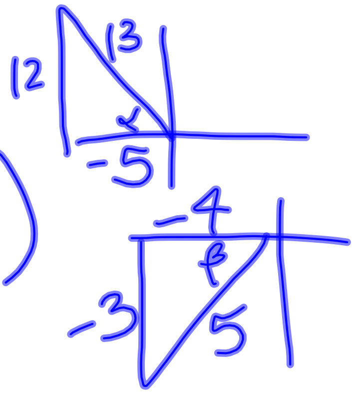
$$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \boxed{\frac{-120}{119}}$$

(−,+)	(+,+)
(−,−)	(+,−)

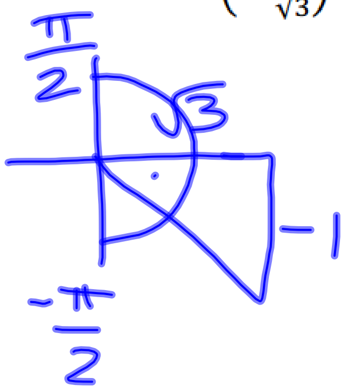
$$\boxed{2\theta \in \text{QIV}}$$

4. Given $\sin\alpha = \frac{12}{13}$, α is in Quadrant II, $\cos\beta = -\frac{4}{5}$, and β is in Quadrant III, find $\sin(\alpha+\beta)$.

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$= \left(\frac{12}{13}\right)\left(\frac{-4}{5}\right) + \left(\frac{-5}{13}\right)\left(\frac{-3}{5}\right)$$
$$= \frac{-48}{65} + \frac{15}{65}$$
$$= \boxed{\frac{-33}{65}}$$

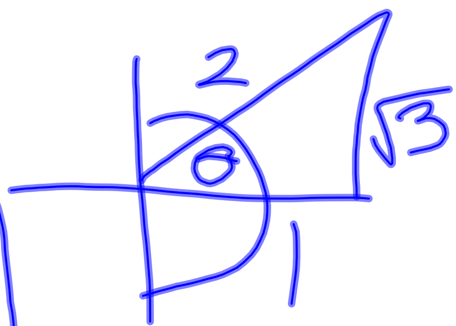


5. Find $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$ exactly in radians.

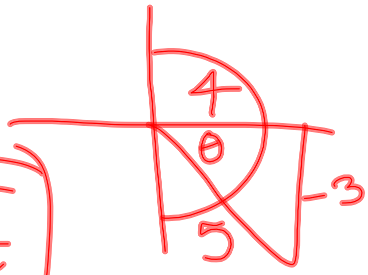


$$= \boxed{-\frac{\pi}{6}}$$

6. Evaluate $\cos\left(\underbrace{\csc^{-1}\frac{2}{\sqrt{3}}}_{\theta}\right) = \boxed{\frac{1}{2}}$

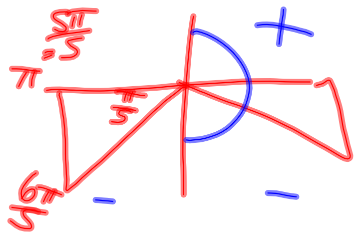


$$\sec\Big(\underbrace{\sin^{-1}\left(-\tfrac{3}{5}\right)}_{\theta}\Big) = \boxed{\frac{5}{4}}$$



$$\sin^{-1}\left(\sin\frac{\pi}{5}\right) = \boxed{\frac{\pi}{5}}$$

$$\sin^{-1}\left(\sin\frac{6\pi}{5}\right) = \boxed{\frac{-\pi}{5}}$$

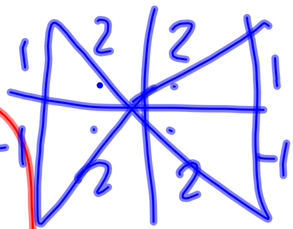


7. Find all solutions (in radians) in the interval $0 \le x < 2\pi$. $\sin^2 x - \frac{1}{4} = 0$

$$\sin^2 x = \frac{1}{4} \Rightarrow \sin x = \pm\sqrt{\frac{1}{4}}$$

$$\sin x = \pm\frac{1}{2}$$

$$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}}$$



8. Find all solutions (in radians) in the interval $0 \leq x < 2\pi$. $2\sin^3 x = \sin x$

$2\sin^3 x - \sin x = 0$

$\sin x(2\sin^2 x - 1) = 0$

$\sin x = 0$

$\boxed{x = 0, \pi}$

$\sin^2 x = \frac{1}{2}$

$\sin x = \pm\frac{1}{\sqrt{2}}$

$\boxed{x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}}$

9. Prove the identity.

$$\frac{1+\cos^2 x}{\sin^2 x} = 2\csc^2 x - 1$$

10. Prove the identity. $\csc x - \cos x \cot x = \sin x$

$$LHS = \frac{1}{\sin x} - \cos x \cdot \frac{\cos x}{\sin x}$$

$$= \frac{1-\cos^2 x}{\sin x}$$

$$= \frac{\sin^2 x}{\sin x}$$

$$= \sin x = RHS$$

Bonus (10 points): Find all solutions (in radians) in the interval $0 \leq x < 2\pi$.

$$\sin 3x + \sin x - \sin 2x = 0$$