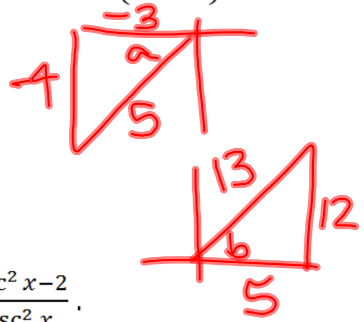
1. Given that $\sin a = -\frac{4}{5}$, $a \in QIII$, and $\tan b = \frac{12}{5}$, $b \in QI$, find $\sin(a-b)$.

$$\sin(a-b) = \sin a \cos b - \cos a \sin b$$
$$= \left(-\frac{4}{5}\right)\left(\frac{5}{13}\right) - \left(-\frac{3}{5}\right)\left(\frac{12}{13}\right)$$
$$= \frac{-20}{65} + \frac{36}{65} = \boxed{\frac{16}{65}}$$



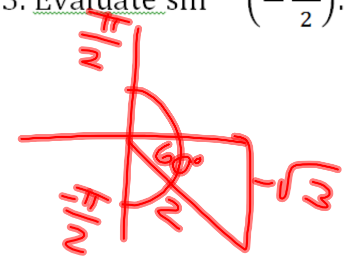
2. Simplify and express as a single trigonometric function $\frac{\csc^2 x - 2}{\csc^2 x}$.

$$\frac{\csc^2 x}{\csc^2 x} - \frac{2}{\csc^2 x} = 1 - \frac{2}{\frac{1}{\sin^2 x}} = 1 - \frac{2}{1} \cdot \frac{\sin^2 x}{1}$$

$$\frac{a-b}{c} = \frac{a}{c} - \frac{b}{c}$$

$$\frac{\frac{1}{\sin^2 x} - 2}{\frac{1}{\sin^2 x}} = \left(\frac{1}{\sin^2 x} - 2\right) \cdot \frac{\sin^2 x}{1} = 1 - 2\sin^2 x = \boxed{\cos 2x}$$

3. Evaluate $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$. Give the answer in radians.



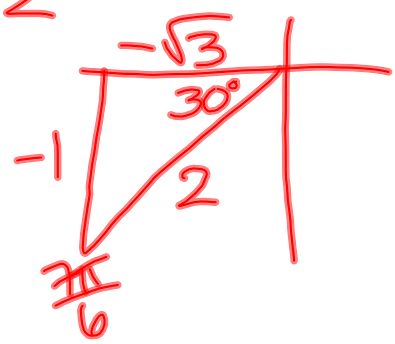
$$= \boxed{-\frac{\pi}{3}}$$

1. Use the half-angle identity to evaluate $\tan\frac{7\pi}{12}$ exactly.

$$\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta}$$

$$\tan\frac{7\pi}{12} = \tan\frac{\frac{7\pi}{12} \cdot 2}{2} = \tan\frac{\left(\frac{7\pi}{6}\right)}{2}$$

$$= \frac{1 - \cos\frac{7\pi}{6}}{\sin\frac{7\pi}{6}} = \frac{1 - \left(\frac{-\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}\right)}$$



$$= \left(1 + \frac{\sqrt{3}}{2}\right)\left(\frac{-2}{1}\right) = \boxed{-2-\sqrt{3}}$$

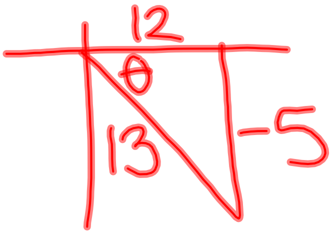
2. Find the exact value of $\cos 212° \cos 122° + \sin 212° \sin 122°$.

$$\cos a \cos b + \sin a \sin b = \cos(a-b)$$
$$= \cos(212° - 122°) = \cos 90° = \boxed{0}$$

3. Find $\sin 2\theta, \cos 2\theta, and \tan 2\theta$ given that $\cos\theta = \frac{12}{13}$ and θ is in Quadrant IV.

$\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(\frac{-5}{13}\right)\left(\frac{12}{13}\right) = \boxed{\frac{-120}{169}}$



$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \left(\frac{12}{13}\right)^2 - \left(\frac{-5}{13}\right)^2 = \frac{144}{169} - \frac{25}{169} = \boxed{\frac{119}{169}}$

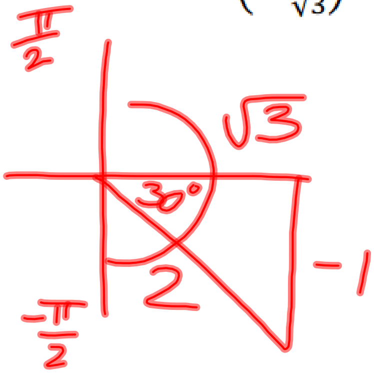
$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \boxed{\frac{-120}{119}}$

(−,+)	(+,+)
(−,−)	(+,−)

In which quadrant does 2θ lie? $2\theta \in$ Q IV

4. Given $\sin\alpha = \frac{12}{13}$, α is in Quadrant II, $\cos\beta = -\frac{4}{5}$, and β is in Quadrant III, find $\sin(\alpha+\beta)$.

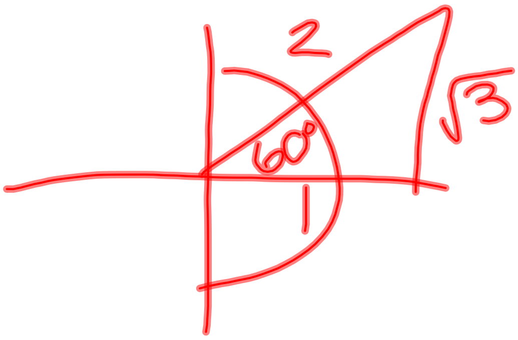
5. Find $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$ exactly in radians.



$\boxed{-\frac{\pi}{6}}$

$\cos 60° =$

6. Evaluate $\cos\left(\underbrace{\csc^{-1}\frac{2}{\sqrt{3}}}_{\theta}\right) = \boxed{\frac{1}{2}}$



$\sec\left(\underbrace{\sin^{-1}\left(-\frac{3}{5}\right)}_{\theta}\right)$

$= \boxed{\frac{5}{4}}$

$\sin^{-1}\left(\sin\frac{\pi}{5}\right) = \frac{\pi}{5}$

$\sin^{-1}\left(\sin\frac{6\pi}{5}\right) = -\frac{\pi}{5}$

7. Find all solutions (in radians) in the interval $0 \leq x < 2\pi$. $\sin^2 x - \frac{1}{4} = 0$

$$\sin^2 x = \frac{1}{4}$$

$$\sin x = \pm\sqrt{\frac{1}{4}} = \pm\frac{1}{2}$$

$$\sin x = \pm\frac{1}{2}$$

$$x = \boxed{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}}$$

8. Find all solutions (in radians) in the interval $0 \le x < 2\pi$. $2\sin^3 x = \sin x$

$2\sin^3 x - \sin x = 0$

$\sin x(2\sin^2 x - 1) = 0$

$\sin x = 0$

$\boxed{x = 0, \pi}$

$\sin^2 x = \frac{1}{2}$

$\sin x = \pm\frac{1}{\sqrt{2}}$

$\boxed{x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}}$

9. Prove the identity.

$$\frac{1+\cos^2 x}{\sin^2 x} = 2\csc^2 x - 1$$

10. Prove the identity. $\csc x - \cos x \cot x = \sin x$

Bonus (10 points): Find all solutions (in radians) in the interval $0 \le x < 2\pi$.

$$\sin 3x + \sin x - \sin 2x = 0$$