Evaluate:

$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$

1. $\csc\frac{5\pi}{4} = -\sqrt{2}$

2. $\cot^{-1}(-1) = \frac{3\pi}{4}$

3. $\cos\frac{3\pi}{8} = \cos\frac{\frac{3\pi}{4}}{2} = +\sqrt{\frac{1+\cos\frac{3\pi}{4}}{2}}$

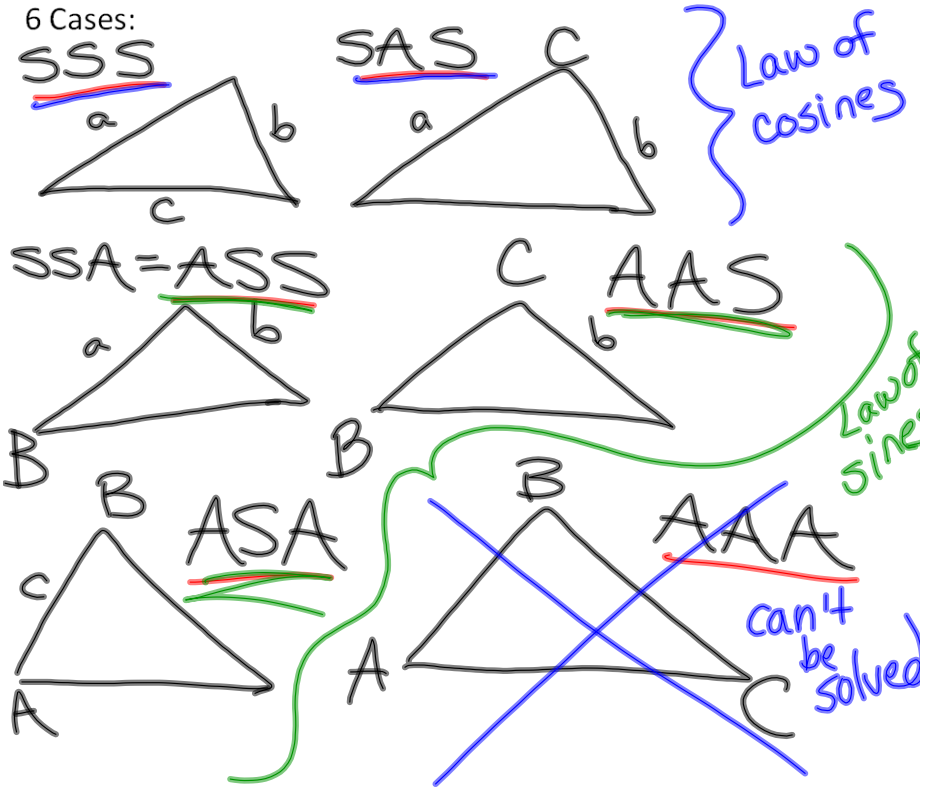
$= \sqrt{\frac{1+\left(-\frac{\sqrt{2}}{2}\right)}{2}} = \sqrt{\left(\frac{2}{2}-\frac{\sqrt{2}}{2}\right)\cdot\frac{1}{2}} = \sqrt{\frac{2-\sqrt{2}}{4}}$

$= \frac{\sqrt{2-\sqrt{2}}}{2}$

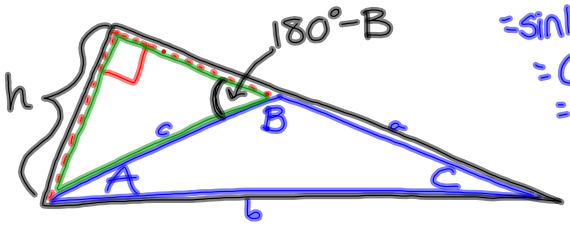
7.1 The Law of Sines

How do we solve oblique (not right) triangles?

6 Cases:



Derivaon of the Law of Sines



$\sin(180°-B) = \sin180°\cos B - \cos180°\sin B = 0\cdot\cos B - (-1)\cdot\sin B = \sin B$

$\sin C = \frac{h}{b}$ (big Δ)

$\sin(180°-B) = \frac{h}{c}$ (small Δ)

$\sin B = \frac{h}{c}$

$h = b\sin C$

$h = c\sin B$

$b\sin C = c\sin B$

$\frac{b\sin C}{bc} = \frac{c\sin B}{bc}$

$\frac{\sin C}{c} = \frac{\sin B}{b}$

$\frac{b\sin C}{\sin B\sin C} = \frac{c\sin B}{\sin B\sin C}$

$\frac{b}{\sin B} = \frac{c}{\sin C}$

The Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

or

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

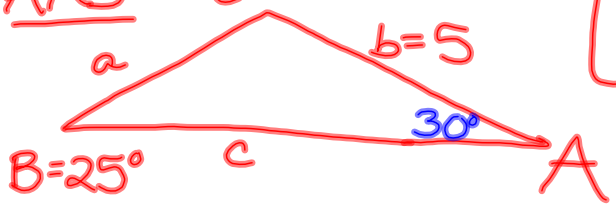
can only be used for AAS, ASA, & ASS

7.1 handout

Solve the triangle.

2. $B=25°$, $C=125°$, $b=5$

AAS



$A=180°-125°-25°$

$=30°$

$\frac{b}{\sin B}=\frac{a}{\sin A}$

$\frac{5}{\sin 25°}=\frac{a}{\sin 30°}$ $\cdot \sin 30°$

$a=\frac{5\sin 30°}{\sin 25°}\approx 5.9$

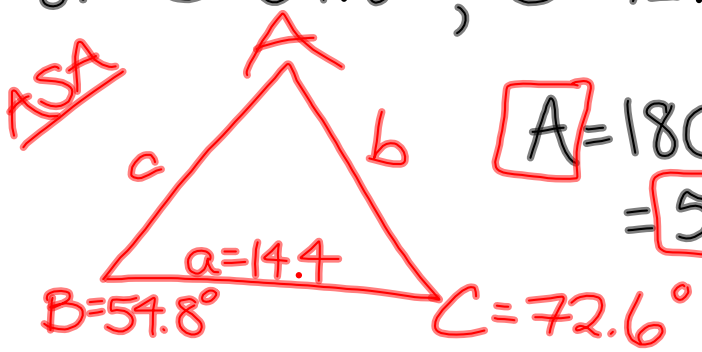
$\frac{b}{\sin B}=\frac{c}{\sin C}$

$\frac{5}{\sin 25°}=\frac{c}{\sin 125°}$

$c=\frac{5\sin 125°}{\sin 25°}\approx 9.7$

8. $B=54.8°$, $C=72.6°$, $a=14.4$

ASA



$A=180°-72.6°-54.8°$

$=52.6°$

$\frac{b}{\sin 54.8°}=\frac{14.4}{\sin 52.6°}$

$b=\frac{14.4\sin 54.8°}{\sin 52.6°}$

≈ 14.8

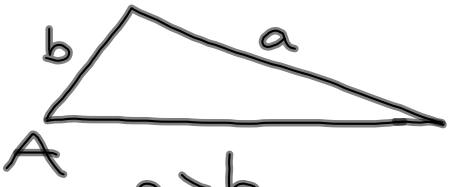
$\frac{c}{\sin 72.6°}=\frac{14.4}{\sin 52.6°}$

$c=\frac{14.4\sin 72.6°}{\sin 52.6°}$

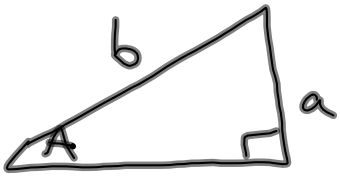
≈ 17.3

ASS, The Problematic Triangle

one solution:



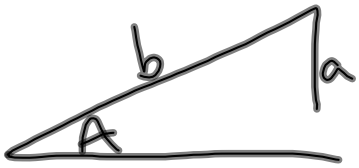
a>b



a<b

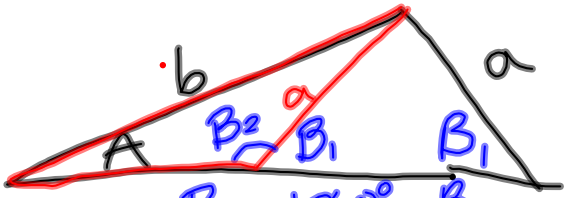
B=90°

no solutions:



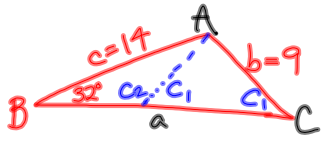
a<b

two solutions: a<b



$B_2 = 180° - B_1$

14. $B = 32°$, $c = 14$, $b = 9$



ASS – 0, 1, or 2 solutions!

$\frac{\sin C}{14} = \frac{\sin 32°}{9}$

$\sin^{-1}(\sin C) = \sin^{-1}\left(\frac{14\sin 32°}{9}\right)$

$C = \sin^{-1}\left(\frac{14\sin 32°}{9}\right)$

$\approx 55.5°$

$A = 180° - B - C$

$= 180° - 32° - 55.5°$

$= 92.5°$

$\frac{a}{\sin 92.5°} = \frac{9}{\sin 32°}$

$a = \frac{9\sin 92.5°}{\sin 32°}$

≈ 17.0

Case 2

$C = 180° - 55.5°$

$= 124.5°$

$A = 180° - B - C_2$

$= 180° - 32° - 124.5°$

$= 23.5°$

$\frac{a}{\sin 23.5°} = \frac{9}{\sin 32°}$

$a = \frac{9\sin 23.5°}{\sin 32°}$

≈ 6.8

HW

7.1 (book)

#1, 2, 4, 6, 7