

Evaluate:

$$1. \csc \frac{5\pi}{4} =$$

$$2. \cot^{-1}(-1) =$$

$$3. \cos \frac{3\pi}{8} =$$

$$1) \csc \frac{5\pi}{4} = \frac{\sqrt{2}}{-1} = -\sqrt{2}$$

$$2) \cot^{-1}(-1) \quad \text{opp} \quad \text{adj} \quad \cot^{-1}(-1) = \frac{\text{adj}}{\text{opp}} = \frac{-1}{1} = -1$$

$$3. \cos \frac{3\pi}{8} = \cos \frac{3\pi}{8} / 2 \quad \cos \frac{\theta}{2} = \pm \sqrt{\frac{1 + \cos \theta}{2}}$$

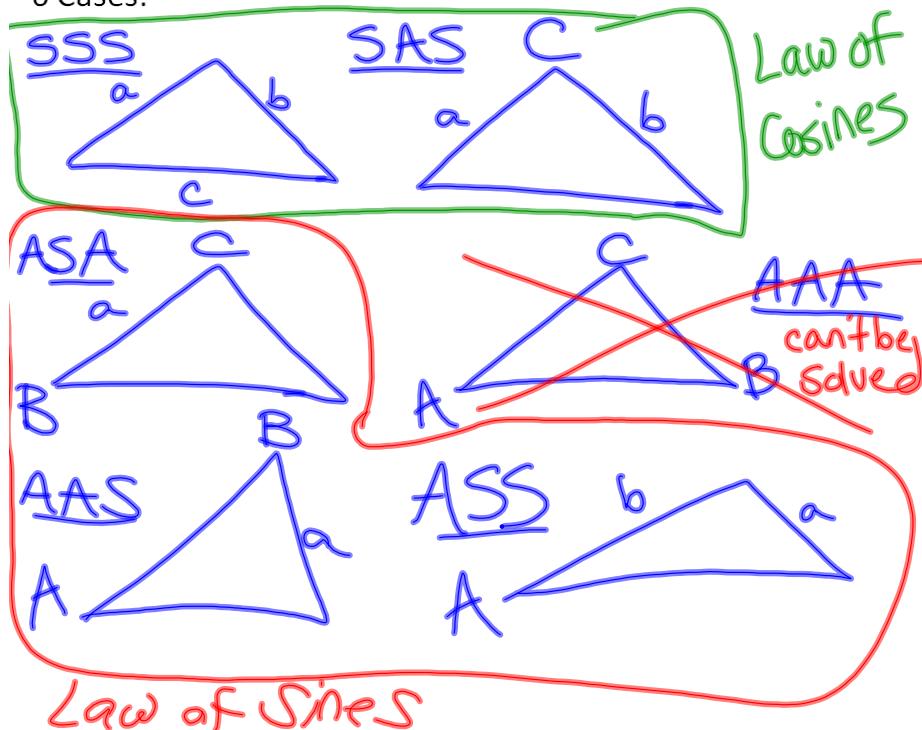
$$= \frac{1 + \cos \frac{3\pi}{4}}{2} = \sqrt{\frac{1 + (-\frac{\sqrt{2}}{2})}{2}} = \sqrt{\left(\frac{1}{2} - \frac{\sqrt{2}}{2}\right) \cdot \frac{1}{2}}$$

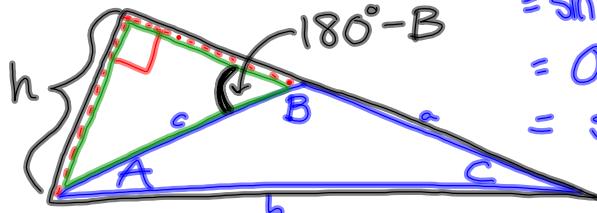
$$= \frac{\sqrt{2} - \sqrt{2}}{2}$$

7.1 The Law of Sines

How do we solve oblique (not right) triangles?

6 Cases:



Derivation of the Law of Sines

$$\begin{aligned}\sin(180^\circ - B) &= \\ &= \sin 180^\circ \cos B - \cos 180^\circ \sin B = \\ &= 0 \cdot \cos B - (-1) \sin B = \\ &= \sin B\end{aligned}$$

$$\sin C = \frac{h}{b} \quad \sin(180^\circ - B) = \frac{h}{c}$$

$$h = b \sin C$$

$$\sin B = \frac{h}{c}$$

$$h = c \sin B$$

$$\frac{b \sin C}{bc} = \frac{c \sin B}{bc} \quad \frac{b \sin C}{\sin B \sin C} = \frac{c \sin B}{\sin B \sin C}$$

$$\frac{\sin C}{c} = \frac{\sin B}{b} \quad \frac{b}{\sin B} = \frac{c}{\sin C}$$

The Law of Sines

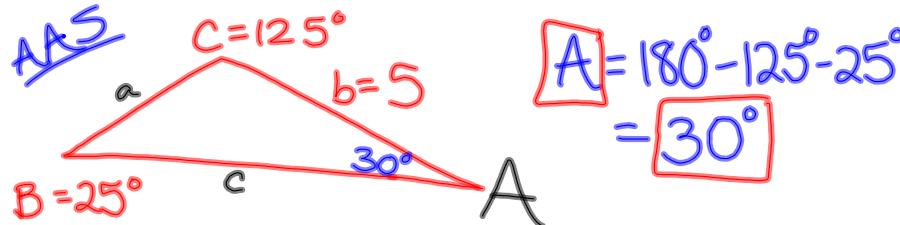
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

or

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

7.1 handout Solve the Triangle.

$$2. \quad B=25^\circ, C=125^\circ, b=5$$



$$\boxed{A=180^\circ - 125^\circ - 25^\circ} = \boxed{30^\circ}$$

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{a}{\sin 30^\circ} = \frac{5}{\sin 25^\circ}$$

$$\boxed{a = \frac{5 \sin 30^\circ}{\sin 25^\circ}}$$

$$\approx \boxed{5.9}$$

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$

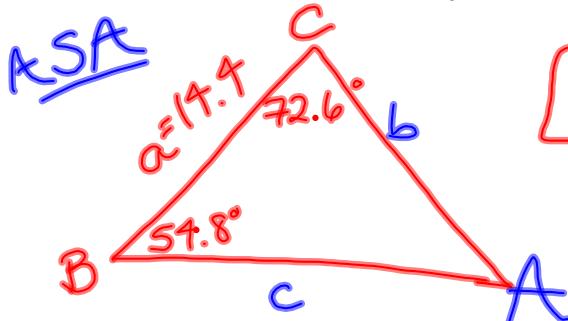
$$\boxed{c = \frac{b \sin C}{\sin B}}$$

$$= \frac{5 \sin 125^\circ}{\sin 25^\circ}$$

$$\approx \boxed{9.7}$$

$$(5 \sin(30)) / (\sin(25))$$

$$8. \quad B=54.8^\circ, C=72.6^\circ, a=14.4$$



$$\boxed{A=180^\circ - 54.8^\circ - 72.6^\circ} = \boxed{52.6^\circ}$$

$$\frac{b}{\sin 54.8^\circ} = \frac{14.4}{\sin 52.6^\circ}$$

$$\boxed{b = \frac{14.4 \sin 54.8^\circ}{\sin 52.6^\circ}}$$

$$\approx \boxed{14.8}$$

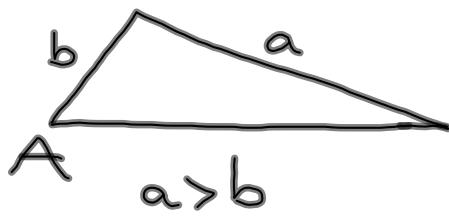
$$\frac{c}{\sin 72.6^\circ} = \frac{14.4}{\sin 52.6^\circ}$$

$$\boxed{c = \frac{14.4 \sin 72.6^\circ}{\sin 52.6^\circ}}$$

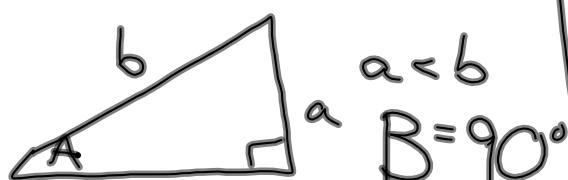
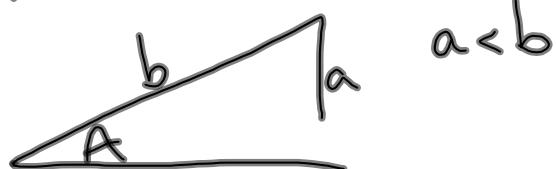
$$\approx \boxed{17.3}$$

ASS, The Problematic Triangle

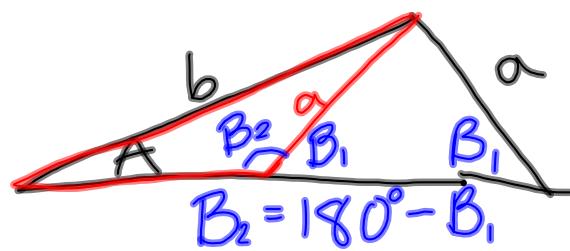
one solution:



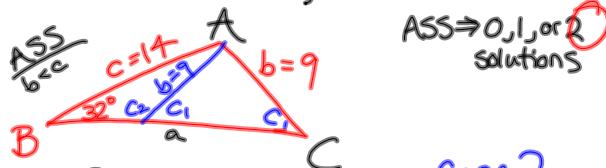
no solutions:



two solutions: $a < b$



$$14. \quad B = 32^\circ, c = 14, b = 9$$



$$\frac{\sin C}{c} = \frac{\sin B}{b}$$

$$\frac{\sin C}{14} = \frac{\sin 32^\circ}{9}$$

$$\sin C = \frac{14 \sin 32^\circ}{9}$$

$$\sin^{-1}(\sin C) = \sin^{-1}\left(\frac{14 \sin 32^\circ}{9}\right)$$

$$C = \sin^{-1}\left(\frac{14 \sin 32^\circ}{9}\right)$$

$$\approx 55.5^\circ$$

$$A = 180^\circ - 55.5^\circ - 32^\circ$$

$$\approx 92.5^\circ$$

$$\frac{a}{\sin 92.5^\circ} = \frac{9}{\sin 32^\circ}$$

$$a = \frac{9 \sin 92.5^\circ}{\sin 32^\circ} \approx 17$$

ASS \Rightarrow 0, 1, or 2 solutions

Case 2

$$C = 180^\circ - 55.5^\circ = 124.5^\circ$$

$$A = 180^\circ - C - B$$

$$= 180^\circ - 124.5^\circ - 32^\circ$$

$$= 23.5^\circ$$

$$\frac{a}{\sin 23.5^\circ} = \frac{9}{\sin 32^\circ}$$

$$a = \frac{9 \sin 23.5^\circ}{\sin 32^\circ} \approx 6.8$$

HW
7.1 (book)

1, 2, 4, 6, 7