

Prove.

$$\frac{1}{1 + \cos x} - \frac{1}{1 - \cos x} = -2 \cot x \csc x$$

$$\begin{aligned} \text{LHS} &= \frac{1}{(1 + \cos x)(1 - \cos x)} - \frac{1}{(1 - \cos x)(1 + \cos x)} \\ &= \frac{1 - \cos x - (1 + \cos x)}{1 - \cos^2 x} = \frac{-2 \cos x}{\sin^2 x} = \\ &= \frac{-2}{1} \cdot \frac{\cos x}{\sin x} \cdot \frac{1}{\sin x} = -2 \cot x \csc x = \text{RHS} \checkmark \end{aligned}$$

Find all solutions (in radians) in the interval $0 \leq x < 2\pi$.

$$\cos 3x + \frac{\sqrt{3}}{2} = 0$$

$$\cos 3x = -\frac{\sqrt{3}}{2}$$

$$3x = \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{17\pi}{6}, \frac{19\pi}{6}, \frac{29\pi}{6}, \frac{31\pi}{6}$$

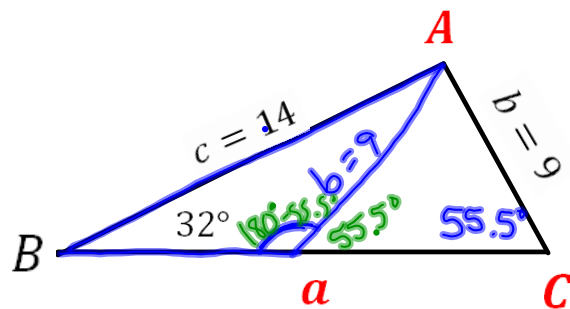
$$x = \frac{5\pi}{18}, \frac{7\pi}{18}, \frac{17\pi}{18}, \frac{19\pi}{18}, \frac{29\pi}{18}, \frac{31\pi}{18}$$

7.1 The Law of Sines, connued

ASS – Problematic Triangle

$$14. B = 32^\circ, c = 14, b = 9$$

Case 1: $C \approx 55.5^\circ, A \approx 92.5^\circ, a \approx 17$

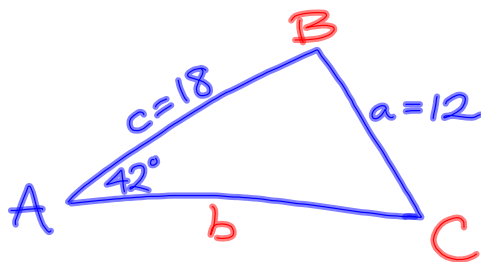


Case 2

bottom right angle

$180^\circ - 1^{\text{st}}$ case bottom right

16. $A = 42^\circ, a = 12, c = 18$



ASS - 0, 1, or 2 solutions

$$18 \cdot \frac{\sin 42^\circ}{12} = \frac{\sin C}{18} \cdot 18$$

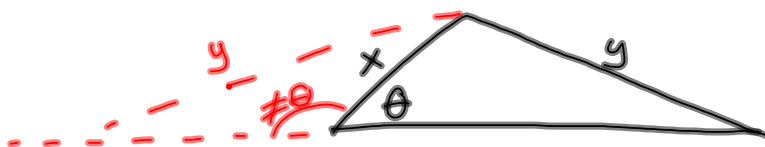
$$\frac{18 \sin 42^\circ}{12} = \sin C$$

$$\sin^{-1}\left(\frac{18 \sin 42^\circ}{12}\right) = C$$

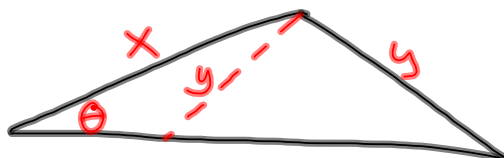
$\sin^{-1}(1.003\dots)$ is undefined
 > 1

no triangle

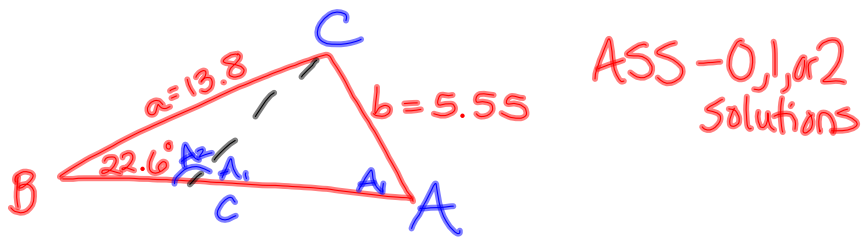
Why does this ASS triangle have only one solution?



The measure of θ and the lengths of x & y are fixed. If we try to reposition y , the measure of θ changes, unlike in the 2-solution case:



18. $B = 22.6^\circ, b = 5.55, a = 13.8$



ASS - 0, 1, or 2 solutions

Case 1

$$\frac{\sin A}{13.8} = \frac{\sin 22.6^\circ}{5.55}$$

$$A = \frac{13.8 \sin 22.6^\circ}{5.55}$$

$$\approx 72.9^\circ$$

$$C = 180^\circ - 22.6^\circ - 72.9^\circ$$

$$= 84.5^\circ$$

$$\frac{c}{\sin 84.5^\circ} = \frac{5.55}{\sin 22.6^\circ}$$

$$c = \frac{5.55 \sin 84.5^\circ}{\sin 22.6^\circ}$$

$$\approx 14.4$$

Case 2

$$A = 180^\circ - 72.9^\circ$$

$$= 107.1^\circ$$

$$C = 180^\circ - B - A$$

$$= 180^\circ - 22.6^\circ - 107.1^\circ$$

$$= 50.3^\circ$$

$$\frac{c}{\sin 50.3^\circ} = \frac{5.55}{\sin 22.6^\circ}$$

$$c = \frac{5.55 \sin 50.3^\circ}{\sin 22.6^\circ} = 11.1$$

7.2 - The Law of Cosines

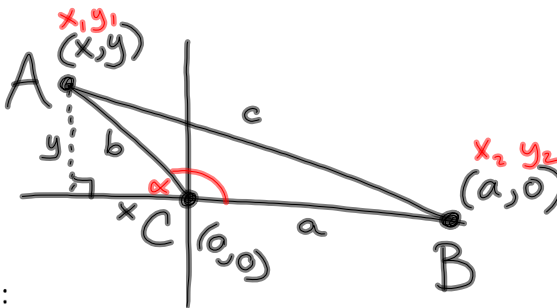
Derivation:

$$\cos C = \frac{x}{b}$$

$$x = b \cos C$$

$$\sin C = \frac{y}{b}$$

$$y = b \sin C$$



Distance Formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$c^2 = (a - x)^2 + (0 - y)^2$$

$$c^2 = (a - b \cos C)^2 + (0 - b \sin C)^2$$

$$c^2 = a^2 - 2ab \cos C + b^2 \cos^2 C + b^2 \sin^2 C$$

$$c^2 = a^2 + b^2 (\sin^2 C + \cos^2 C) - 2ab \cos C$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

The Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cdot \cos B$$

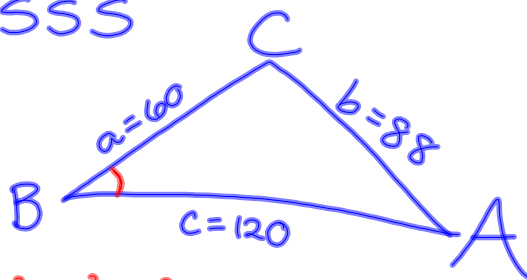
$$c^2 = a^2 + b^2 - 2ab \cdot \cos C$$

used to solve SSS & SAS

7.2 Handout:

16. $a = 60, b = 88, c = 120$. $B = ?$

SSS

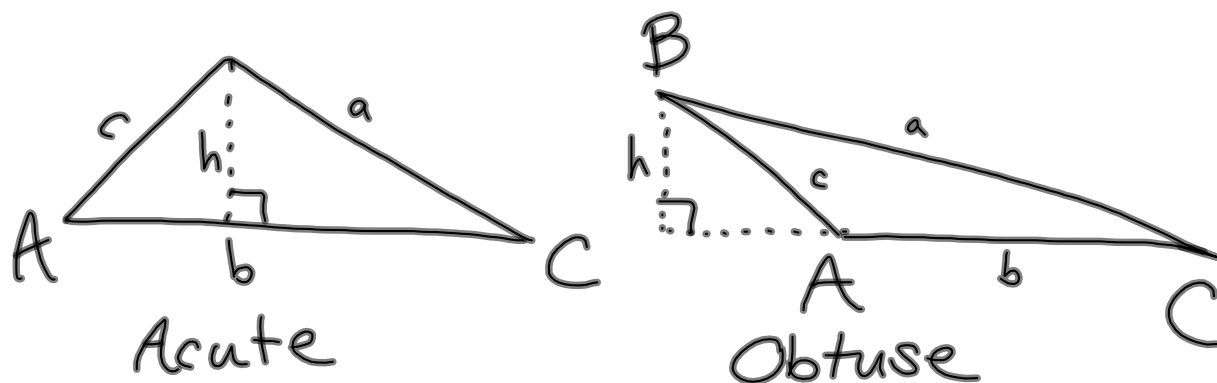


$$b^2 = a^2 + c^2 - 2ac \cdot \cos B$$

$$\frac{2ac \cdot \cos B}{2ac} = \frac{a^2 + c^2 - b^2}{2ac}$$

$$B = \cos^{-1} \left(\frac{a^2 + c^2 - b^2}{2ac} \right) = \cos^{-1} \left(\frac{60^2 + 120^2 - 88^2}{2(60)(120)} \right)$$

$$= \cos^{-1} \left(\frac{(60^2 + 120^2 - 88^2)}{(2 * 60 * 120)} \right) = \boxed{44.6^\circ}$$

7.1/7.2 Area of a Triangle

Find the area of the triangle.

$$A = 50^\circ, b = 13 \text{ cm}, c = 6 \text{ cm}$$

Handout Homework:

7.1 #13-21 odd

7.2 #9-19 odd, ~~25-29 odd~~