Prove.

$$\frac{1}{1+\cos x}-\frac{1}{1-\cos x}=-2\cot x\csc x$$

$$LHS=\frac{1}{(1+\cos x)}\cdot\frac{1-\cos x}{(1-\cos x)}-\frac{1}{(1-\cos x)}\cdot\frac{1+\cos x}{(1+\cos x)}$$

$$=\frac{1-\cos x-(1+\cos x)}{1-\cos^2 x}=\frac{-2\cos x}{\sin^2 x}=$$

$$=\frac{-2}{1}\cdot\frac{\cos x}{\sin x}\cdot\frac{1}{\sin x}=-2\cot x\csc x=RHS\checkmark$$

Find all solutions (in radians) in the interval $0\le x<2\pi$.

$$\cos 3x+\frac{\sqrt{3}}{2}=0$$

$$\cos 3x=-\frac{\sqrt{3}}{2}$$

$$3x=\frac{5\pi}{6},\frac{7\pi}{6},\frac{17\pi}{6},\frac{19\pi}{6},\frac{29\pi}{6},\frac{31\pi}{6}$$

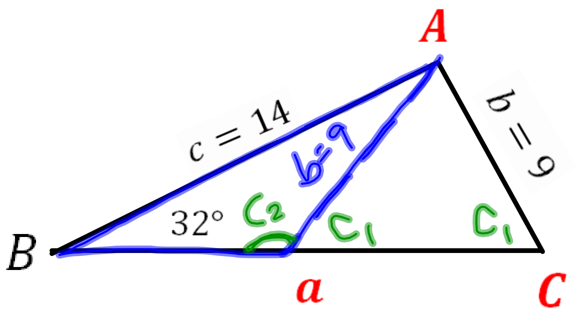
$$x=\frac{5\pi}{18},\frac{7\pi}{18},\frac{17\pi}{18},\frac{19\pi}{18},\frac{29\pi}{18},\frac{31\pi}{18}$$

7.1 The Law of Sines, connued

ASS – Problematic Triangle

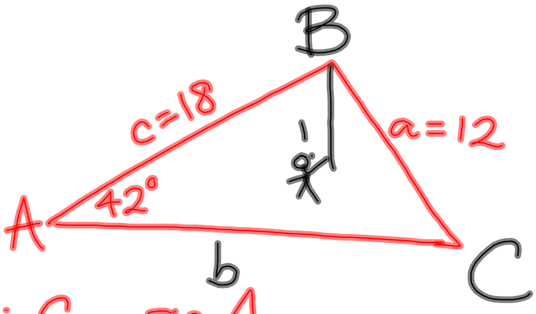
14. $B=32°$, $c=14$, $b=9$

Case 1: $C\approx 55.5°$, $A\approx 92.5°$, $a\approx 17$



Case 2 bottom right ∠ is supplement of case 1

16. $A = 42°, a = 12, c = 18$



ASS - 0, 1, or 2 solutions

$\frac{\sin C}{c} = \frac{\sin A}{a}$

$18 \cdot \frac{\sin C}{18} = \frac{\sin 42°}{12} \cdot 18$

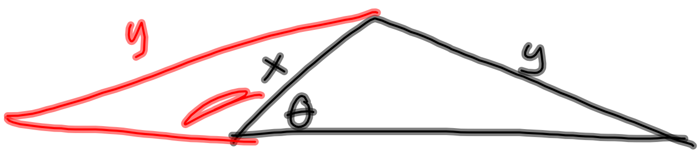
$\sin C = \frac{18 \sin 42°}{12}$

$C = \sin^{-1}\left(\frac{18 \sin 42°}{12}\right)$

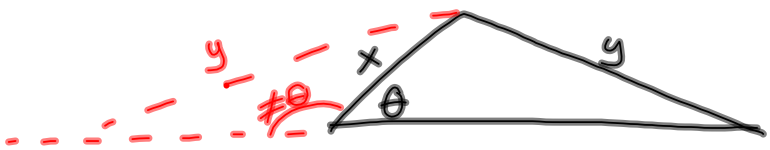
$= \sin^{-1}(1.003\ldots)$ is undefined

$\Rightarrow$ there is no triangle

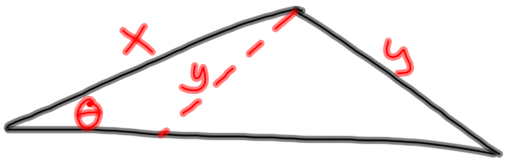
Why does this ASS triangle have only one solution?



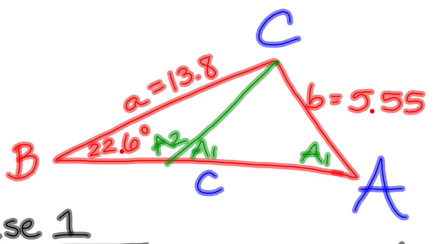
Why does this ASS triangle have only one solution?



The measure of θ and the lengths of x & y are fixed. If we try to reposition y, the measure of θ changes, unlike in the 2-solution case:



18. $B = 22.6°, b = 5.55, a = 13.8$



ASS - 0, 1, or 2 solutions

case 1

$$\frac{\sin A}{13.8} = \frac{\sin 22.6°}{5.55}$$

$$\sin A = \frac{13.8 \sin 22.6°}{5.55}$$

$$A = \sin^{-1}\left(\frac{13 \sin 22.6°}{5.55}\right)$$

$$\approx \boxed{72.9°}$$

$$C = 180° - 72.9° - 22.6°$$

$$= \boxed{84.5°}$$

$$\frac{c}{\sin 84.5°} = \frac{5.55}{\sin 22.6°}$$

$$c = \frac{5.55 \sin 84.5°}{\sin 22.6°} = \boxed{14.4}$$

case 2

$$A = 180° - 72.9°$$

$$= \boxed{107.1°}$$

$$C = 180° - A - B$$

$$= 180° - 107.1° - 22.6$$

$$= \boxed{50.3°}$$

$$\frac{c}{\sin 50.3°} = \frac{5.55}{\sin 22.6°}$$

$$c = \frac{5.55 \sin 50.3°}{\sin 22.6°} = \boxed{11.1}$$

7.2 - The Law of Cosines

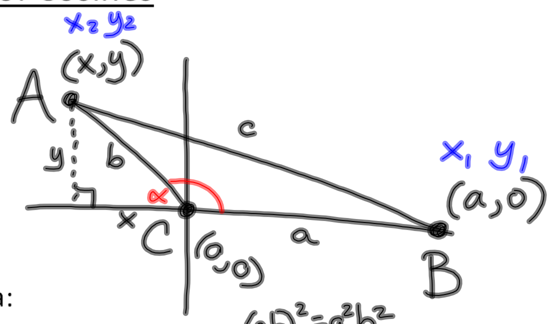
Derivation:

$\cos C = \frac{x}{b}$

$x = b\cos C$

$\sin C = \frac{y}{b}$

$y = b \cdot \sin C$



Distance Formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$(ab)^2 = a^2b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$c^2 = (x-a)^2 + (y-0)^2 = (b \cdot \cos C - a)^2 + (b \cdot \sin C)^2$$

$$= (b^2\cos^2 C - 2ab \cdot \cos C + a^2) + b^2 \sin^2 C$$

$$= a^2 + b^2(\underbrace{\sin^2 C + \cos^2 C}_{=1}) - 2ab\cos C$$

$$c^2 = a^2 + b^2 - 2ab \cdot \cos C$$

The Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

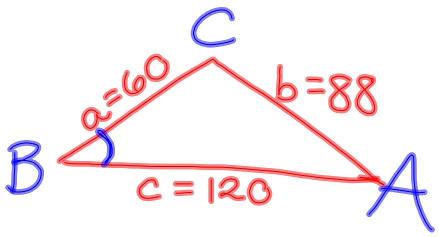
$$b^2 = a^2 + c^2 - 2ac \cdot \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cdot \cos C$$

used for SSS & SAS

7.2 Handout:

16. $a = 60, b = 88, c = 120.\ B = ?$



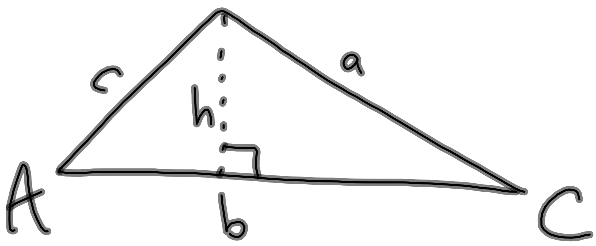
$$b^2 = a^2 + c^2 - 2ac\cdot\cos B$$

$$\frac{\cancel{2ac}\cdot\cos B}{\cancel{2ac}} = \frac{a^2+c^2-b^2}{2ac}$$

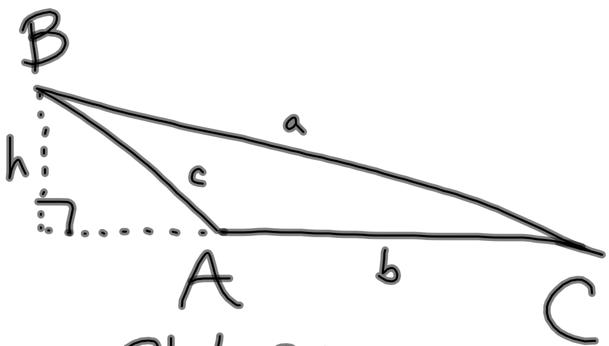
$$B = \cos^{-1}\left(\frac{a^2+c^2-b^2}{2ac}\right) = \cos^{-1}\left(\frac{60^2+120^2-88^2}{2(60)(120)}\right)$$

$$= \cos^{-1}\left((60\text{^}2+120\text{^}2-88\text{^}2)/(2*60*120)\right) \approx \boxed{44.6^\circ}$$

7.1/7.2 Area of a Triangle



Acute



Obtuse

Find the area of the triangle.

$A = 50° , b = 13\ cm , c = 6\ cm$

Handout Homework:

7.1 #13-21 odd

7.2 #9-19 odd, ~~25-29 odd~~