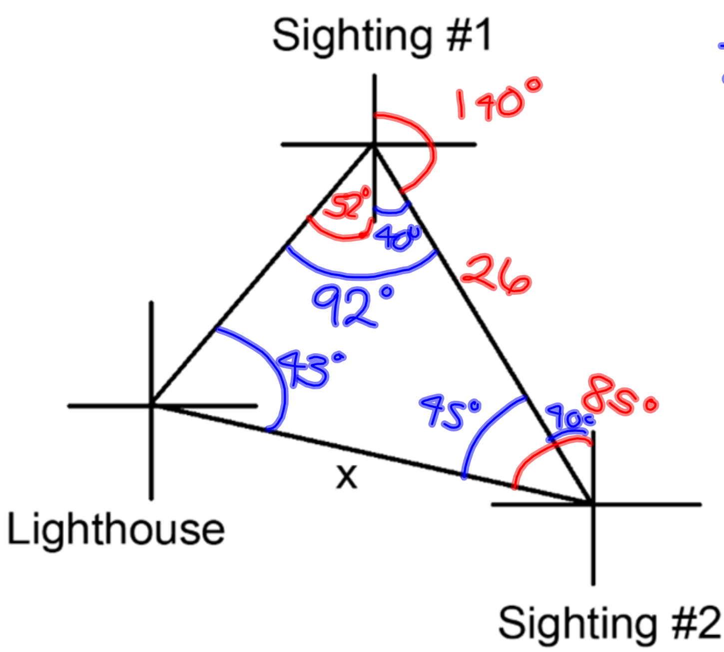
7. A navigator on a ship sights a lighthouse at a bearing of $S52°W$. After traveling 26 miles at a heading of $140°$, the ship sights the lighthouse at a bearing of $N85°W$. How far is the ship from the lighthouse at the second sighting?

Hint: recall that heading is measured clockwise from north, and that $S52°W$ is read as "52 degrees west of south."

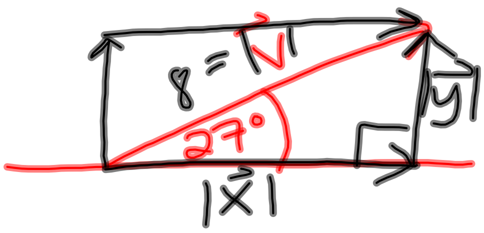
Drawing is not necessarily to scale.



$$\frac{x}{\sin 92°} = \frac{26}{\sin 43°}$$

$$x = \boxed{38.1 \text{ mi}}$$

8. Find the magnitude of the horizontal and vertical components of a vector with magnitude 8 pounds pointed in a direction of $27°$ above the horizontal.



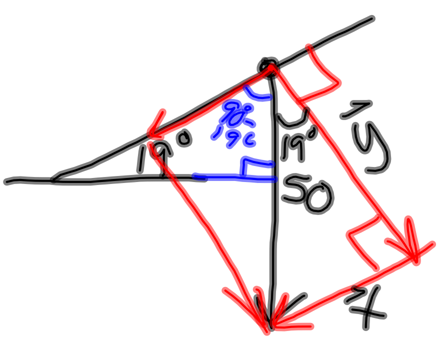
horizontal: $\cos 27° = \frac{|\vec{x}|}{8}$

$|\vec{x}| = 8\cos 27° = \boxed{7.1 \text{ lb}}$

vertical: $\sin 27° = \frac{|\vec{y}|}{8}$

$|\vec{y}| = 8\sin 27° = \boxed{3.6 \text{ lb}}$

9. A 50-pound object rests on a ramp that is inclined $19°$. Find the magnitude of the components of the force parallel to and perpendicular (normal) to the ramp to the nearest tenth of a pound.

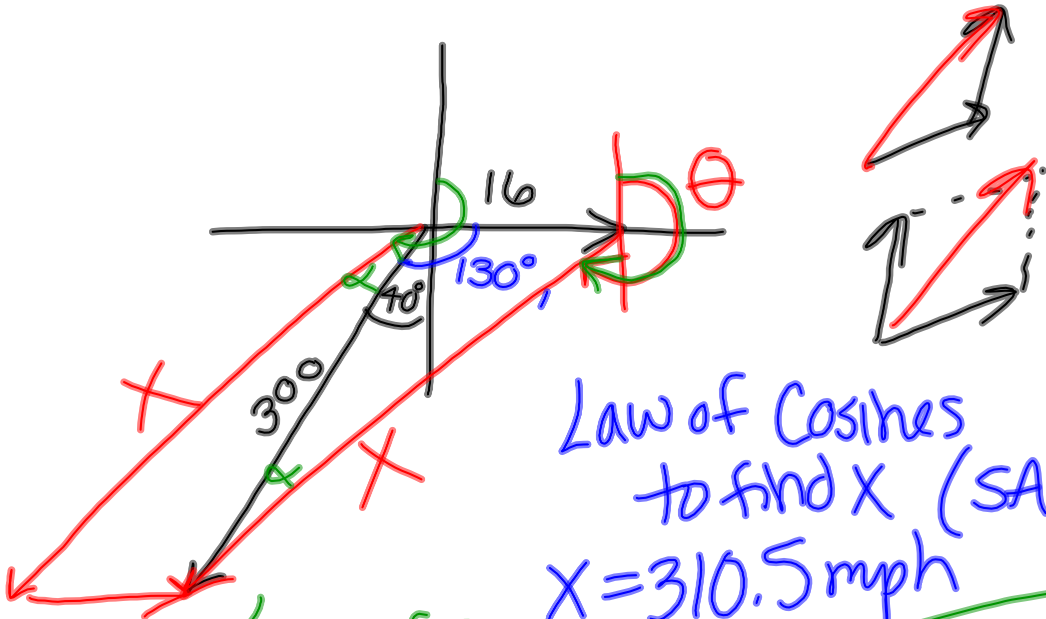


$\sin 19° = \frac{|\vec{x}|}{50}$

$|\vec{x}| = \boxed{16.3 \text{ lb parallel}}$

$\cos 19° = \frac{|\vec{y}|}{50}$ $|\vec{y}| = 47.3$ lb perpendicular

Bonus: An airplane travels with an airspeed of 300 miles per hour at a heading of 220°. The wind is blowing from the west at a speed of 16 miles per hour. Find the direction and speed that the plane should attempt in order to accommodate for the wind to actually achieve an airspeed of 300 miles per hour and heading of 220°.



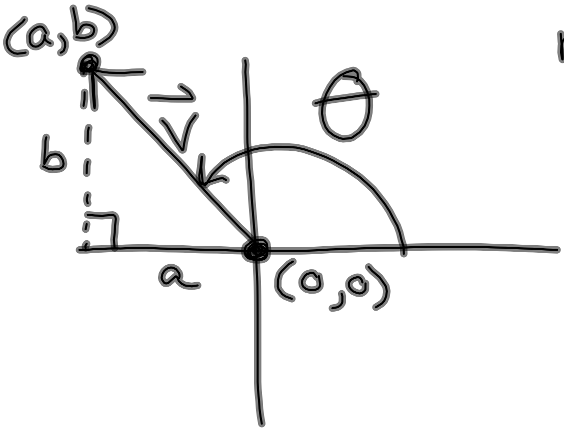
Law of Cosines to find x (SAS)

$x = 310.5$ mph

Law of Sines to find α

$\theta = \boxed{222.3°}$

A vector is unique up to its length (magnitude) & direction angle.



magnitude $|\vec{v}| = \sqrt{a^2+b^2}$

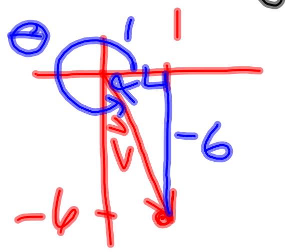
direction angle θ is measured counter-clockwise from 0°

Determine the magnitude and direction angle of $\vec{v} = \langle 1, -6 \rangle$.

magnitude: $|\vec{v}| = \sqrt{a^2+b^2}$

$|\vec{v}| = \sqrt{1^2+(-6)^2} = \boxed{\sqrt{37}}$

direction angle: measured counter-clockwise from positive x-axis



$\vec{v} \in Q\text{IV} \Rightarrow \theta$ is between $270°$ & $360°$

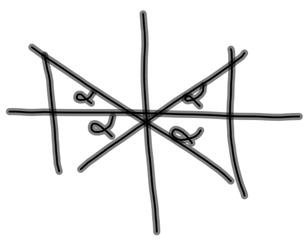
$\tan\alpha = \left|\frac{-6}{1}\right| \quad \alpha = \tan^{-1}|-6| = 80.5°$

$\theta = 360° - 80.5° = \boxed{279.5°}$

magnitude of $\vec{v} = \langle a, b \rangle$ is

$$|\vec{v}| = \sqrt{a^2+b^2}$$

direction angle θ is found by reference angle $\alpha = \tan^{-1}\left|\frac{b}{a}\right|$



If $\theta \in Q\text{I}$, $\theta = \alpha$

II, $\theta = 180° - \alpha$

III, $\theta = 180° + \alpha$

IV, $\theta = 360° - \alpha$

Vector operations

$\vec{v} = \langle a,b \rangle$; $\vec{w} = \langle c,d \rangle$; $k \in \mathbb{R}$

k is a real number, not a vector

1. $|\vec{v}| = \sqrt{a^2+b^2}$
2. "scalar multiplication" $k\vec{v} = k\langle a,b \rangle = \langle ka, kb \rangle$
3. $\vec{v}+\vec{w} = \langle a+c, b+d \rangle$
4. $-\vec{v} = \langle -a,-b \rangle$ (same vector, pointing in opposite direction)
5. $\vec{v} - \vec{w} = \langle a-c, b-d \rangle$
6. $\vec{0} = \langle 0,0 \rangle$ "zero vector"

$\vec{v} = \langle 12,-5 \rangle$; $\vec{w} = \langle 2,7 \rangle$

a. $|\vec{v}| = \sqrt{12^2+(-5)^2} = \boxed{13}$

b. $\vec{v}+\vec{w} = \langle 12+2, -5+7 \rangle = \boxed{\langle 14,2 \rangle}$

c. $-5\vec{v} = -5\langle 12,-5 \rangle = \langle -5(12), -5(-5) \rangle = \boxed{\langle -60, 25 \rangle}$

d. $3\vec{v} - 4\vec{w}$

$= 3\langle 12,-5 \rangle - 4\langle 2,7 \rangle$

$= \langle 36,-15 \rangle - \langle 8,28 \rangle$

$= \langle 36-8, -15-28 \rangle = \boxed{\langle 28,-43 \rangle}$

Vector Multiplication

$\vec{V} \cdot \vec{W}$ vs. $\vec{V} \times \vec{W}$

"dot product"

result is a scalar (real #, not a vector)

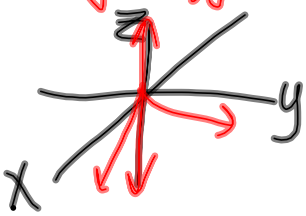
$\vec{V} = \langle x_1, y_1 \rangle$

$\vec{W} = \langle x_2, y_2 \rangle$

$\vec{V} \cdot \vec{W} = x_1 x_2 + y_1 y_2$

"cross product"

result is a vector perpendicular to the plane spanned by $\vec{V}$ & $\vec{W}$.



$\vec{V}_1 = \langle 1, 2 \rangle$; $\vec{V}_2 = \langle -3, 4 \rangle$; $\vec{V}_3 = \langle 5, -6 \rangle$

$\vec{V}_1 \cdot \vec{V}_2 = 1(-3) + 2(4) = -3 + 8 = \boxed{5}$

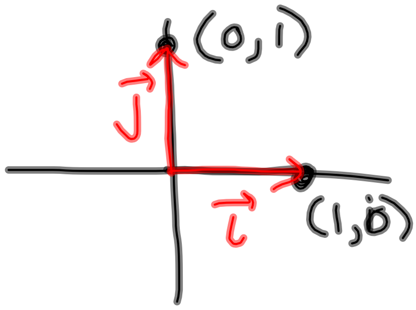
$\vec{V}_1 \cdot \langle \vec{V}_2 + \vec{V}_3 \rangle = \langle 1, 2 \rangle \cdot \langle -3+5, 4+(-6) \rangle$

$= \langle 1, 2 \rangle \cdot \langle 2, -2 \rangle$

$= 1(2) + 2(-2) = 2 - 4 = \boxed{-2}$

7.6, cont. - Unit Vectors

A **unit vector** is a vector whose magnitude is 1.



Special Unit Vectors:

$\vec{\boldsymbol{i}} = \langle 1,0 \rangle$ & $\vec{\boldsymbol{j}} = \langle 0,1 \rangle$

In 3 dimensions we would have

$\vec{\boldsymbol{i}} = \langle 1,0,0 \rangle$, $\vec{\boldsymbol{j}} = \langle 0,1,0 \rangle$ & $\vec{\boldsymbol{k}} = \langle 0,0,1 \rangle$

Any vector given in component form can also be written in terms of $\vec{i}$ and $\vec{j}$.

$$\underbrace{\langle a,b \rangle}_{\text{component form}} = \langle a,0 \rangle + \langle 0,b \rangle$$

$$= a\langle 1,0 \rangle + b\langle 0,1 \rangle$$

$$= \underbrace{a\vec{i} + b\vec{j}}_{\text{in terms of } \vec{i} \,\&\, \vec{j}}$$

Vector operations for vectors given in terms of $\vec{i}$ and $\vec{j}$ are in some ways simpler than for vectors given in component form, as $\vec{i}$ and $\vec{j}$ can be treated like variables.

7.6 #46 & 48. $\vec{u} = 2\vec{i} + \vec{j}$; $\vec{v} = -3\vec{i} - 10\vec{j}$; $\vec{w} = \vec{i} - 5\vec{j}$

46. $\vec{v} + 3\vec{w} = -3\vec{i} - 10\vec{j} + 3(\vec{i} - 5\vec{j})$

$= -3\vec{i} - 10\vec{j} + 3\vec{i} - 15\vec{j}$

$= -25\vec{j} = \langle 0, -25 \rangle$

48. $(\vec{u} - \vec{v}) + \vec{w} = 2\vec{i} + \vec{j} - (-3\vec{i} - 10\vec{j}) + \vec{i} - 5\vec{j}$

$= 2\vec{i} + \vec{j} + 3\vec{i} + 10\vec{j} + \vec{i} - 5\vec{j}$

$= 6\vec{i} + 6\vec{j} = \langle 6, 6 \rangle$

Note that the magnitude of a vector given in the form $\vec{v} = a\vec{i} + b\vec{j}$ is still found by the formula $|\vec{v}| = \sqrt{a^2 + b^2}$

$\vec{v} = 3\vec{i} - 2\vec{j}$

$|\vec{v}| = \sqrt{3^2 + (-2)^2} = \sqrt{9+4} = \boxed{\sqrt{13}}$

$\cancel{\sqrt{(3\vec{i})^2 + (-2\vec{j})^2}}$

$\cancel{i^2 = -1}$ $\vec{i} \neq i = \sqrt{-1}$

Given a vector $\vec{v} = < a, b >$, we can find a **<u>unit vector $\vec{u}$ in the direction of $\vec{v}$</u>** by dividing each component (a & b) by the magnitude $|\vec{v}|$.

Given $\vec{v} = < -3,4 >$, find a unit vector $\vec{u}$ in the direction of $\vec{v}$.

Homework:

7.6 #9-26 all