

Expand and simplify: $\cos(90^\circ - \theta)$
 $= \cos 90^\circ \cos \theta + \sin 90^\circ \sin \theta$
 $= 0 \cdot \cos \theta + 1 \cdot \sin \theta$
 $= \boxed{\sin \theta}$

$f^2(x) = [f(x)]^2$

$\cos(a-b) = \cos a \cos b + \sin a \sin b$
 $\cos 2x = \cos^2 x - \sin^2 x$
 $= 2\cos^2 x - 1$
 $= 1 - 2\sin^2 x$
 $\sin 2x = 2\sin x \cos x$

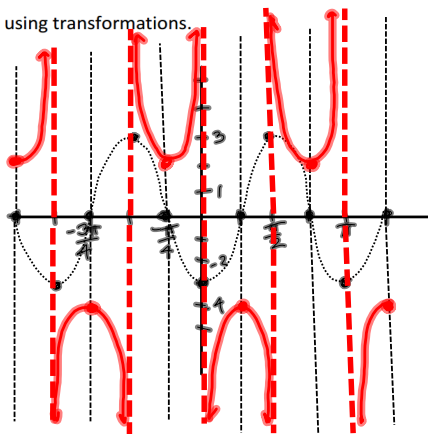
Find all solutions (in radians) in the interval $0 \leq x < 2\pi$. $2 \sin^2 2x = 1 - \cos 2x$

$2(\sin 2x)^2 = 1 - \cos 2x$
 $2(2\sin x \cos x)^2 = 1 - \cos 2x$
 $2(4\sin^2 x \cos^2 x) = 1 - \cos 2x$
 $8\sin^2 x \cos^2 x = 1 - \cos 2x$
 $8\sin^2 x \cos^2 x = 1 - (1 - 2\sin^2 x)$
 $8\sin^2 x \cos^2 x = 2\sin^2 x$
 $8\sin^2 x \cos^2 x - 2\sin^2 x = 0$

$2\sin^2 x (4\cos^2 x - 1) = 0$
 $2\sin^2 x = 0$ $4\cos^2 x - 1 = 0$
 $\sin^2 x = 0$ $\cos^2 x = \frac{1}{4}$
 $\sin x = 0$ $\cos x = \pm \frac{1}{2}$
 $x = 0, \pi$ $x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

Graph $y = -3 \sec\left(2x + \frac{3\pi}{2}\right) - 1$ using transformations.

"amp": 3
 period: $\frac{2\pi}{2} = \pi$
 h. shift: left $\frac{3\pi}{2} = \frac{3\pi}{4}$
 v. shift: down 1



Homework questions?

$\vec{u} = \langle 5, -2 \rangle$, $\vec{v} = \langle -4, 7 \rangle$

7.6

17. $|3\vec{u}| - |\vec{v}|$

$|\vec{v}| = \sqrt{a^2 + b^2}$

$= | \langle 15, -6 \rangle | - | \langle -4, 7 \rangle |$

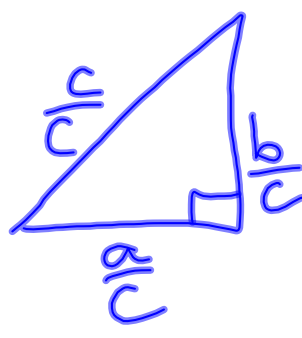
$= \sqrt{15^2 + (-6)^2} - \sqrt{(-4)^2 + 7^2}$

$= \boxed{\sqrt{261} - \sqrt{65}}$

Given a vector $\vec{v} = \langle a, b \rangle$, we can find a **unit vector \vec{u} in the direction of \vec{v}** by dividing each component (a & b) by the magnitude $|\vec{v}|$.

$$\vec{u} = \frac{\vec{v}}{|\vec{v}|}$$

$$|\vec{v}| = \sqrt{a^2 + b^2}$$

$$= \left\langle \frac{a}{\sqrt{a^2 + b^2}}, \frac{b}{\sqrt{a^2 + b^2}} \right\rangle$$


Given $\vec{v} = \langle -3, 4 \rangle$, find a unit vector \vec{u} in the direction of \vec{v} .

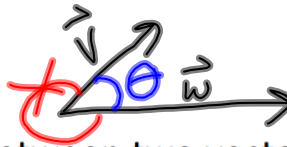
$$|\vec{v}| = \sqrt{(-3)^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

$$\vec{u} = \frac{\vec{v}}{|\vec{v}|} = \left\langle \frac{-3}{5}, \frac{4}{5} \right\rangle$$

verify:

$$|\vec{u}| = \sqrt{\left(\frac{-3}{5}\right)^2 + \left(\frac{4}{5}\right)^2} = \sqrt{\frac{9}{25} + \frac{16}{25}} = \sqrt{\frac{25}{25}} = \sqrt{1} = 1$$

7.6 Vectors, cont.



The smallest nonnegative angle between two vectors is given by

$$\cos \theta = \frac{\vec{v} \cdot \vec{w}}{|\vec{v}| |\vec{w}|}, \text{ or equivalently, } \boxed{\theta = \cos^{-1} \frac{\vec{v} \cdot \vec{w}}{|\vec{v}| |\vec{w}|}}.$$

Recall that both the dot product and magnitude vector operations yield a scalar (non-vector real number) quantity, so we can find the inverse cosine value.

$$\text{Given } \vec{v} = \langle a, b \rangle \text{ and } \vec{w} = \langle c, d \rangle,$$

$$|\vec{v}| = \sqrt{a^2 + b^2}$$

$$\vec{v} \cdot \vec{w} = ac + bd$$

7.6 #64. Given $\vec{a} = \langle -3, -3 \rangle$ and $\vec{b} = \langle -5, 2 \rangle$, find the smallest non-negative angle between \vec{a} and \vec{b} .

$$\cos \theta = \frac{\vec{v}_1 \cdot \vec{v}_2}{|\vec{v}_1| |\vec{v}_2|} = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|}$$

$$\vec{a} \cdot \vec{b} = (-3)(-5) + (-3)(2) = 15 - 6 = 9$$

$$|\vec{a}| = \sqrt{(-3)^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$|\vec{b}| = \sqrt{(-5)^2 + 2^2} = \sqrt{25+4} = \sqrt{29}$$

$$\theta = \cos^{-1} \left(\frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} \right) = \cos^{-1} \left(\frac{9}{3\sqrt{2} \cdot \sqrt{29}} \right) = \boxed{66.8^\circ}$$

$$\cos^{-1} \left(9 / (3 \sqrt{2} \sqrt{29}) \right)$$

$$68. \vec{u} = 3\vec{i} + 2\vec{j} ; \vec{v} = -\vec{i} + 4\vec{j}$$

$$= \langle 3, 2 \rangle \quad = \langle -1, 4 \rangle$$

$$\vec{u} \cdot \vec{v} = 3(-1) + 2(4) = -3 + 8 = 5$$

$$|\vec{u}| = \sqrt{3^2 + 2^2} = \sqrt{9 + 4} = \sqrt{13}$$

$$|\vec{v}| = \sqrt{(-1)^2 + 4^2} = \sqrt{1 + 16} = \sqrt{17}$$

$$\theta = \cos^{-1} \left(\frac{\vec{u} \cdot \vec{v}}{|\vec{u}| |\vec{v}|} \right) = \cos^{-1} \left(\frac{5}{\sqrt{13} \sqrt{17}} \right) = \boxed{70.3^\circ}$$

Verify the identity. Show all steps.

HW:

7.6 #

33-47 } odd
53-57 }
63-67 }

$$\cos^a 5x \cos^b 3x + \sin^a 5x \sin^b 3x = 1 - 2\sin^2 x$$

$$\text{LHS} = \cos(5x - 3x)$$

$$= \cos 2x$$

$$= 1 - 2\sin^2 x$$

$$= \text{RHS} \quad \square$$