

Given  $\vec{v} = \langle -1, 6 \rangle$ ,  $\vec{w} = \langle 6, 1 \rangle$

1. Find  $2\vec{v} - 3\vec{w}$ .

$$2\langle -1, 6 \rangle - 3\langle 6, 1 \rangle = \langle -2, 12 \rangle - \langle 18, 3 \rangle = \boxed{\langle -20, 9 \rangle}$$

2. Find  $|\vec{v}|$ .

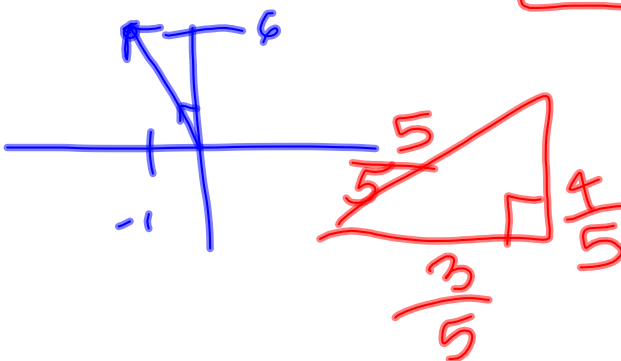
$$\sqrt{(-1)^2 + 6^2} = \sqrt{37}$$

3. Find  $|\vec{w}|$ .

$$\sqrt{6^2 + 1^2} = \sqrt{37}$$

4. Find  $\vec{v} \cdot \vec{w}$ .

$$(-1)(6) + 6(1) = 0$$



5. Find the angle  $\theta$  between  $\vec{v}$  and  $\vec{w}$ .

$$\theta = \cos^{-1} \left( \frac{\vec{v} \cdot \vec{w}}{|\vec{v}| |\vec{w}|} \right) = \cos^{-1} \left( \frac{0}{\sqrt{37} \sqrt{37}} \right) = \cos^{-1}(0) = 90^\circ$$

6. Find a unit vector  $\vec{u}$  in the same direction as  $\vec{v}$ .

$$\left\langle \frac{-1}{\sqrt{37}}, \frac{6}{\sqrt{37}} \right\rangle = \left\langle \frac{-\sqrt{37}}{37}, \frac{6\sqrt{37}}{37} \right\rangle$$

7.6

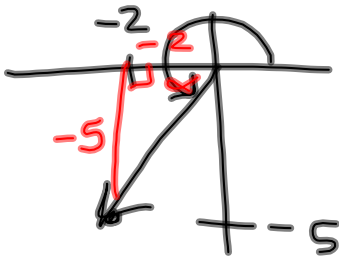
37.  $\vec{r} = \langle -2, -8 \rangle$

$$|\vec{r}| = \sqrt{(-2)^2 + (-8)^2} = \sqrt{4 + 64} = \sqrt{68} = 2\sqrt{17}$$

$$\vec{u} = \left\langle \frac{-2}{2\sqrt{17}}, \frac{-8}{2\sqrt{17}} \right\rangle$$

$$= \left\langle \frac{-1}{\sqrt{17}}, \frac{-4}{\sqrt{17}} \right\rangle = \left\langle \frac{-\sqrt{17}}{17}, \frac{-4\sqrt{17}}{17} \right\rangle$$

$$53. \vec{u} = \langle -2, -5 \rangle$$



$$\tan \alpha = \left| \frac{b}{a} \right|$$

$$\alpha = \tan^{-1} \left| \frac{-5}{-2} \right| = 68.2^\circ$$

$$\theta = \alpha + 180^\circ = \boxed{248.2^\circ}$$

Find the radius (in cm) of a circle that contains an arc of length 8m, subtending a  $100^\circ$  angle.

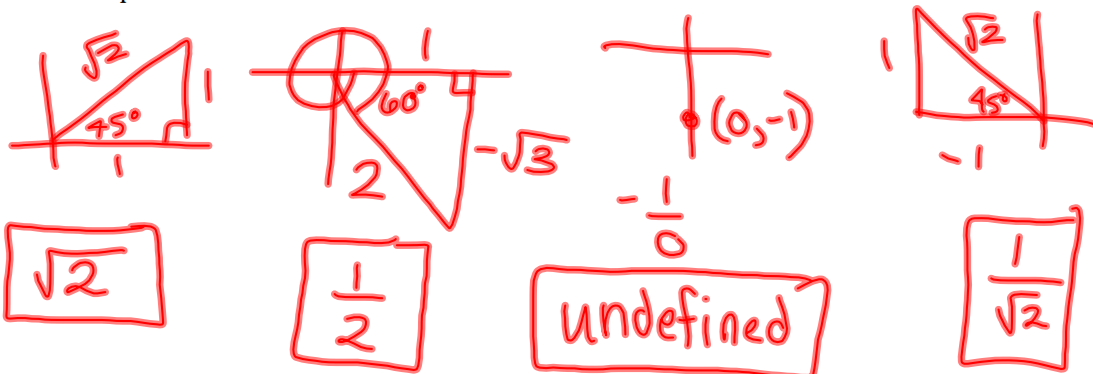
$$s = 8m ; \theta = 100^\circ ; r = ? \text{ cm}$$

$$\frac{s}{\theta} = \frac{r\theta}{\theta} \quad r = \frac{s}{\theta}$$

$$r = \frac{8m}{100^\circ} \cdot \frac{100 \text{ cm}}{1m} \cdot \frac{180^\circ}{\pi} = \boxed{\frac{1440 \text{ cm}}{\pi}}$$

Evaluate the following trigonometric expressions:

1.  $\sec \frac{\pi}{4}$       2.  $\cos(-420^\circ)$       3.  $\tan(-90^\circ)$       4.  $\sin 135^\circ$



Solve for x. (all solutions, no restrictions)

$\sin(2x) - \sin(x) = 0$   
 $2\sin x \cos x - \sin x = 0$   
 $\sin x (2\cos x - 1) = 0$   
 $\sin x = 0$        $\cos x = \frac{1}{2}$

$x = 0 + 2\pi k$   
 $x = \pi + 2\pi k$

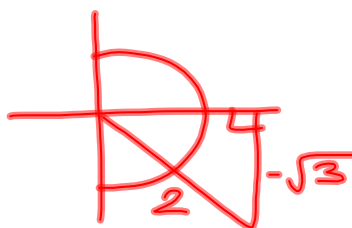
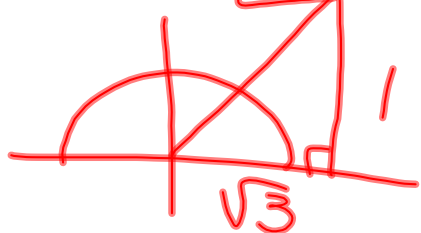
$x = \frac{\pi}{3} + 2\pi k$   
 $x = \frac{5\pi}{3} + 2\pi k$

$x = \pi k$

Evaluate the inverse functions. Give your answers in radians.

1.  $\csc^{-1}\left(-\frac{2}{\sqrt{3}}\right) = -\frac{\pi}{3}$

2.  $\cot^{-1}(\sqrt{3}) = \frac{\pi}{6}$



$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ :

sin  
csc  
tan

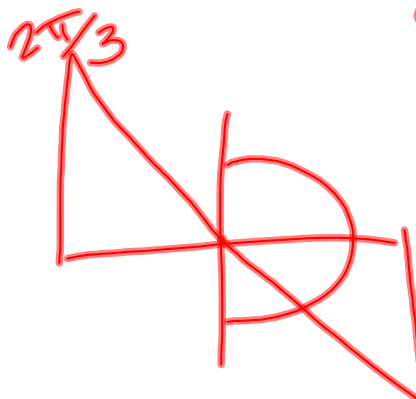
$(0, \pi)$ :

cos  
sec  
cot

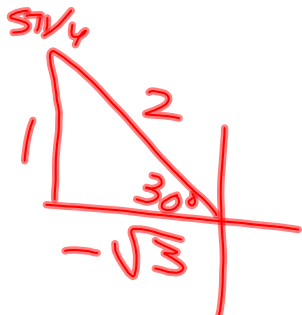
Evaluate.

1.  $\sin^{-1}\left(\sin \frac{\pi}{5}\right) = \frac{\pi}{5}$

2.  $\tan^{-1}\left(\tan \frac{2\pi}{3}\right) = -\frac{\pi}{3}$



Given that  $\tan \frac{x}{2} = \frac{1 - \cos x}{\sin x}$ , evaluate  $\tan \frac{5\pi}{12}$ .

$$\begin{aligned} \tan \frac{5\pi}{12} &= \frac{1 - \cos \frac{5\pi}{6}}{\sin \frac{5\pi}{6}} = \frac{1 - \left(-\frac{\sqrt{3}}{2}\right)}{\frac{1}{2}} \\ &= \left(1 + \frac{\sqrt{3}}{2}\right) \cdot \frac{2}{1} \\ &= \boxed{2 + \sqrt{3}} \end{aligned}$$


Prove.

$$\sin 3x = 3 \sin x - 4 \sin^3 x$$

$$\text{LHS} = \sin(2x + x) =$$

$$\begin{aligned} &= \sin 2x \cos x + \cos 2x \sin x = \\ &= (2 \sin x \cos x) \cos x + (\cos^2 x - \sin^2 x) \sin x = \\ &= 2 \sin x \cos^2 x + \sin x \cos^2 x - \sin^3 x = \\ &= 3 \sin x \cos^2 x - \sin^3 x = \\ &= 3 \sin x (1 - \sin^2 x) - \sin^3 x = \\ &= 3 \sin x - 3 \sin^3 x - \sin^3 x = \\ &= 3 \sin x - 4 \sin^3 x = \text{RHS} \quad \square \end{aligned}$$

Find the exact value of  $\cos \frac{\pi}{12} \cos \frac{\pi}{4} - \sin \frac{\pi}{12} \sin \frac{\pi}{4}$ .

$$= \cos \left( \frac{\pi}{12} + \frac{\pi}{4} \right) =$$

$$= \cos \left( \frac{\pi}{12} + \frac{3\pi}{12} \right) = \cos \frac{4\pi}{12} =$$

$$= \cos \frac{\pi}{3} = \boxed{\frac{1}{2}}$$

$$f(x) = -3 \csc \left( 4x + \frac{\pi}{2} \right) - 2$$

- "amplitude"
- period
- horizontal shift
- vertical shift

$$f(x) = \frac{2}{3} \cot \left( 3x - \frac{3\pi}{4} \right) + 1$$

- "amplitude"
- period
- horizontal shift
- vertical shift

