Assignments for the break:

- Read 6.1-6.3
- **memorize your identities!!!**

After the break:

- 6.2 #1-41 odd
- 6.1 #1-69 odd (proofs)
- 6.3 #1-24 all; 30-36 all; 49-93 odd

Chapter 6 - Trigonometric Identities and Equations

Reciprocal Identities

$\csc x = \frac{1}{\sin x}$, $\sin x = \frac{1}{\csc x}$

$\sec x = \frac{1}{\cos x}$, $\cos x = \frac{1}{\sec x}$

$\cot x = \frac{1}{\tan x}$, $\tan x = \frac{1}{\cot x}$

Ratio Identities

$\tan x = \frac{\sin x}{\cos x}$, $\cot x = \frac{\cos x}{\sin x}$

Cofunction Identities

$\sin\left(\frac{\pi}{2} - x\right) = \cos x$, $\cos\left(\frac{\pi}{2} - x\right) = \sin x$

$\tan\left(\frac{\pi}{2} - x\right) = \cot x$, $\cot\left(\frac{\pi}{2} - x\right) = \tan x$

$\csc\left(\frac{\pi}{2} - x\right) = \sec x$, $\sec\left(\frac{\pi}{2} - x\right) = \csc x$

Odd-Even Identities

$\cos(-x) = \cos x$, $\sin(-x) = -\sin x$, $\tan(-x) = -\tan x$

$\sec(-x) = \sec x$, $\csc(-x) = -\csc x$, $\cot(-x) = -\cot x$

Pythagorean Identities

$\sin^2 x + \cos^2 x = 1$

$1 + \cot^2 x = \csc^2 x$

$\tan^2 x + 1 = \sec^2 x$

$(a+b)^2 \neq a^2 + b^2$

$(a+b)(a+b)$

Useful formulas from Algebra:

$a^2 - b^2 = (a-b)(a+b)$

$(a+b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

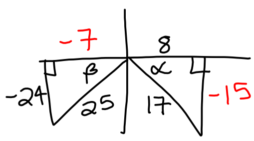
$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

6.2 - Sum and Difference Identities

$$\sin(a+b) = \sin a \cos b + \cos a \sin b$$
$$\sin(a-b) = \sin a \cos b - \cos a \sin b$$
$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$
$$\cos(a-b) = \cos a \cos b + \sin a \sin b$$
$$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$$
$$\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$$

Given $\cos\alpha = \frac{8}{17}$, $\alpha \in Q\,IV$
$\sin\beta = \frac{-24}{25}$, $\beta \in Q\,III$
find $\sin(\alpha+\beta)$, $\cos(\alpha+\beta)$, $\tan(\alpha+\beta)$, & determine the quadrant in which $\alpha+\beta$ lies.



*Pythagorean triples that are useful to know:
3,4,5 ; 5,12,13 ; 7,24,25 ;
& 8,15,17

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$= \left(\frac{-15}{17}\right)\left(\frac{-7}{25}\right) + \left(\frac{8}{17}\right)\left(\frac{-24}{25}\right)$$
$$= \frac{105 - 192}{425} = \boxed{\frac{-87}{425}}$$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$= \left(\frac{8}{17}\right)\left(\frac{-7}{25}\right) - \left(\frac{-15}{17}\right)\left(\frac{-24}{25}\right)$$
$$= \frac{-56 - 360}{425} = \boxed{\frac{-416}{425}}$$

$$\tan(\alpha+\beta) = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = \boxed{\frac{87}{416}}$$

$\alpha+\beta \in Q\,III$

Cofunction Identities

The function of an angle is equal to the cofunction of its complement.

θ & $90° - \theta$ or θ & $\frac{\pi}{2} - \theta$

are complementary angles

$\cos\left(\frac{\pi}{2} - x\right) = \cos\frac{\pi}{2}\cos x + \sin\frac{\pi}{2}\sin x$

$= 0 \cdot \cos x + 1 \cdot \sin x = \sin x$

Double-Angle Identities

$\sin 2\theta = \sin(\theta + \theta)$

$= \sin\theta\cos\theta + \cos\theta\sin\theta$

$\boxed{\sin 2\theta = 2\sin\theta\cos\theta}$

"The sine of twice any angle is equal to two times the sine of that angle times the cosine of that angle."

$\sin(42x) = \sin[2(21x)] = 2\sin 21x\cos 21x$

$\sin(8x) = 2\sin 4x\cos 4x$

$\sin 2(4x)$

$= 2(2\sin 2x\cos 2x)\cos 4x$

$$\boxed{\sin 2\theta = 2\sin\theta\cos\theta}$$

The sine of twice any angle is equal to two times the sine of that angle times the cosine of that angle.

$$\sin 6\theta = 2\sin 3\theta\cos 3\theta$$

$$\cancel{\sin 8\theta =}$$

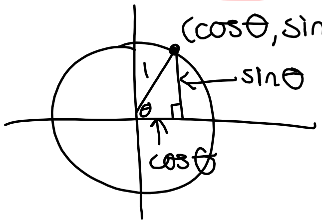
$$\sin 44\theta = 2\sin 22\theta\cos 22\theta$$

$$\sin 3\theta = 2\sin\tfrac{3}{2}\theta\cos\tfrac{3}{2}\theta$$
$$= \sin(2\theta+\theta)$$
$$= \sin 2\theta\cos\theta + \cos 2\theta\sin\theta$$

$$\cos 2\theta = \cos(\theta+\theta)$$
$$= \cos\theta\cos\theta - \sin\theta\sin\theta$$
$$= (\cos\theta)^2 - (\sin\theta)^2$$

$$\boxed{\cos 2\theta = \cos^2\theta - \sin^2\theta}$$

$$\boxed{\begin{gathered}\sin\theta^2 = \sin(\theta^2)\\ \neq \\ (\sin\theta)^2 = \sin^2\theta\end{gathered}}$$



$$(\sin\theta)^2 + (\cos\theta)^2 = 1^2$$

$$\boxed{\sin^2\theta + \cos^2\theta = 1}$$ Pythagorean Identity

$$\sin^2\theta = 1 - \cos^2\theta$$
$$\cos^2\theta = 1 - \sin^2\theta$$

$$\cos 2\theta = (1-\sin^2\theta) - \sin^2\theta$$

$$\boxed{\cos 2\theta = 1 - 2\sin^2\theta}$$

$$\cos 2\theta = \cos^2\theta - (1-\cos^2\theta)$$

$$\boxed{\cos 2\theta = 2\cos^2\theta - 1}$$

$$\tan 2\theta = \tan(\theta + \theta)$$

$$= \frac{\tan\theta + \tan\theta}{1 - \tan\theta\tan\theta}$$

$$\boxed{\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}}$$

The other two Pythagorean Identities are derived from the first.

$$\frac{\sin^2 x + \cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$$

$$\frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$\boxed{\tan^2 x + 1 = \sec^2 x}$$

$$\frac{\sin^2 x + \cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x}$$

$$\frac{\sin^2 x}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x}$$

$$\boxed{1 + \cot^2 x = \csc^2 x}$$