- 6.2 #1-41 odd - due Mon. 01/09
- 6.1 #1-69 odd (proofs) - due Tues. 01/17
- 6.3 #1-24 all; 30-36 all; 49-93 odd

Chapter 6 - Trigonometric Identities and Equations

Reciprocal Identities

$\csc x = \frac{1}{\sin x}$, $\sin x = \frac{1}{\csc x}$

$\sec x = \frac{1}{\cos x}$, $\cos x = \frac{1}{\sec x}$

$\cot x = \frac{1}{\tan x}$, $\tan x = \frac{1}{\cot x}$

Ratio Identities

$\tan x = \frac{\sin x}{\cos x}$, $\cot x = \frac{\cos x}{\sin x}$

Cofunction Identities

$\sin\left(\frac{\pi}{2} - x\right) = \cos x$, $\cos\left(\frac{\pi}{2} - x\right) = \sin x$

$\tan\left(\frac{\pi}{2} - x\right) = \cot x$, $\cot\left(\frac{\pi}{2} - x\right) = \tan x$

$\csc\left(\frac{\pi}{2} - x\right) = \sec x$, $\sec\left(\frac{\pi}{2} - x\right) = \csc x$

Odd-Even Identities

$\cos(-x) = \cos x$, $\sin(-x) = -\sin x$, $\tan(-x) = -\tan x$

$\sec(-x) = \sec x$, $\csc(-x) = -\csc x$, $\cot(-x) = -\cot x$

Pythagorean Identities

$\sin^2 x + \cos^2 x = 1$

$1 + \cot^2 x = \csc^2 x$

$\tan^2 x + 1 = \sec^2 x$

$(a+b)^2 \neq a^2 + b^2$

$(a+b)(a+b)$

Useful formulas from Algebra:

$a^2 - b^2 = (a-b)(a+b)$

$(a+b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

Sum and Difference Identities

$\sin(a+b) = \sin a \cos b + \cos a \sin b$

$\sin(a-b) = \sin a \cos b - \cos a \sin b$

$\cos(a+b) = \cos a \cos b - \sin a \sin b$

$\cos(a-b) = \cos a \cos b + \sin a \sin b$

$\tan(a+b) = \dfrac{\tan a + \tan b}{1 - \tan a \tan b}$

$\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$

Double-Angle Identities

$\sin 2x = 2 \sin x \cos x$

$$\begin{aligned} \cos 2x &= \cos^2 x - \sin^2 x \\ &= 2\cos^2 x - 1 \\ &= 1 - 2\sin^2 x \end{aligned}$$

$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$

Given $\sin\theta = -\frac{2}{3}$, $\theta \in QIII$,

Find $\sin 2\theta$, $\cos 2\theta$, $\tan 2\theta$, and the quadrant in which 2θ lies.

Double-Angle Identities

$\sin 2x = 2 \sin x \cos x$

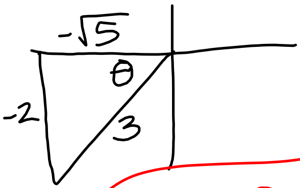
$$\begin{aligned} \cos 2x &= \cos^2 x - \sin^2 x \\ &= 2\cos^2 x - 1 \\ &= 1 - 2\sin^2 x \end{aligned}$$

$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$

$\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(-\frac{2}{3}\right)\left(-\frac{\sqrt{5}}{3}\right)$

$\boxed{\sin 2\theta = \frac{4\sqrt{5}}{9}}$



$2\theta \in QI$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \left(\frac{-\sqrt{5}}{3}\right)^2 - \left(\frac{-2}{3}\right)^2$

$= \frac{5}{9} - \frac{4}{9} = \boxed{\frac{1}{9} = \cos 2\theta}$

$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{4\sqrt{5}/9}{1/9} = \boxed{4\sqrt{5} = \tan 2\theta}$

$= \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2 \cdot \frac{2\sqrt{5}}{5}}{1 - \left(\frac{2}{\sqrt{5}}\right)^2} = \frac{\frac{4\sqrt{5}}{5}}{\frac{5}{5} - \frac{4}{5}} = \frac{\frac{4\sqrt{5}}{5}}{1/5} = \boxed{4\sqrt{5}}$

Half-Angle Identities

$\sin\frac{x}{2} = ?$

$\cos 2\theta = 1 - 2\sin^2\theta$

Let $\theta = \frac{x}{2}$.

$\cos 2\left(\frac{x}{2}\right) = 1 - 2\sin^2\left(\frac{x}{2}\right)$

$\cos x = 1 - 2\sin^2\frac{x}{2}$

$2\sin^2\frac{x}{2} = 1 - \cos x$

$\sin^2\frac{x}{2} = \frac{1-\cos x}{2}$

$\boxed{\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}}$

$\cos\frac{x}{2} = ?$

$\cos 2\theta = 2\cos^2\theta - 1$

Let $\theta = \frac{x}{2}$

$\cos \cancel{2}\left(\frac{x}{\cancel{2}}\right) = 2\cos^2\left(\frac{x}{2}\right) - 1$

$\cos x + 1 = 2\cos^2\frac{x}{2}$

$\frac{\cos x + 1}{2} = \cos^2\frac{x}{2}$

$\boxed{\pm\sqrt{\frac{\cos x + 1}{2}} = \cos\frac{x}{2}}$

Half-Angle Identities

$$\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}} \quad , \quad \cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$$

$$\tan\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{1+\cos x}}$$

$$= \frac{\sin x}{1+\cos x}$$

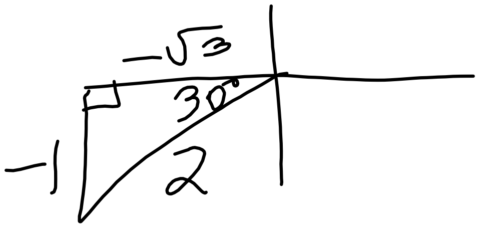
$$\boxed{\tan\frac{x}{2} = \frac{1-\cos x}{\sin x}}$$

$$\tan\frac{7\pi}{12} = \tan\frac{7\pi/6}{2}$$

$$\frac{\cancel{2}}{1}\cdot\frac{7\pi}{\underset{6}{\cancel{12}}} = \frac{\theta}{\cancel{2}}\cdot\frac{\cancel{2}}{1}$$

$$\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta}$$

$$= \frac{1-\cos 7\pi/6}{\sin 7\pi/6}$$



$$= \frac{1-\left(-\sqrt{3}/2\right)}{-1/2}$$

$$= \left(1+\frac{\sqrt{3}}{2}\right)\cdot\frac{-2}{1}$$

$$= \boxed{-2-\sqrt{3}}$$

6.3 Evaluate using the half-angle identity.

14. $\sin(112.5^\circ) = \sin\frac{225^\circ}{2}$

$\in$ QII

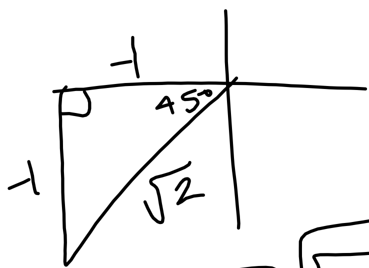
$\sin 112.5^\circ > 0$

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\theta = 225^\circ$$

$$= +\sqrt{\frac{1-\cos 225^\circ}{2}}$$

$$= \sqrt{\frac{\frac{2}{2}1-\left(-\sqrt{2}/2\right)}{2}}$$



$$= \sqrt{\frac{\frac{2}{2}+\frac{\sqrt{2}}{2}}{\frac{2}{1}}} = \sqrt{\frac{2+\sqrt{2}}{2}\cdot\frac{1}{2}} = \sqrt{\frac{2+\sqrt{2}}{4}} = \boxed{\frac{\sqrt{2+\sqrt{2}}}{2}}$$

To prove an identity means to show that one side of the identity can be rewritten in a form that is identical to the other side.

There is no one method that works for every identity, the following are some helpful guidelines:

- remember that you are trying to prove that this equation is true, so you can't treat it like an equation -- no working on both sides (e.g. you can't add something to both sides, or divide both sides by something). You must start with one side and rewrite it until it is equal to the other side. It is okay to meet in the middle if you get stuck and must work from both sides.
- if one side is more complicated than the other, start with the more complicated side and try to simplify it
- use rules of algebra to find common denominators, add fractions, square binomials, factor, multiply by a form of 1, add 0, etc.
- apply known identities to rewrite parts of an expression in a more useful form, e.g. since $\sin^2 x + \cos^2 x = 1$, you can replace the expression $\sin^2 x + \cos^2 x$ with 1.
- when in doubt, rewrite in terms of sine and cosine

6.3 Prove/Verify the identity.

50. $\underline{\cos 8x} = \cos^2 4x - \sin^2 4x$

$LHS = \cos 8x = \cos[2(4x)] =$

$= \cos^2 4x - \sin^2 4x = RHS$

52. $\dfrac{\cos 2x}{\sin^2 x} = \cot^2 x - 1$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$
$= 1 - 2\sin^2\theta$
$= 2\cos^2\theta - 1$

$\dfrac{a \pm b}{c} = \dfrac{a}{c} \pm \dfrac{b}{c}$

$LHS = \dfrac{\cos 2x}{\sin^2 x}$

$= \dfrac{\cos^2 x - \sin^2 x}{\sin^2 x}$

$= \dfrac{\cos^2 x}{\sin^2 x} - \dfrac{\sin^2 x}{\sin^2 x} = \cot^2 x - 1 = RHS$ ✓

54. $\dfrac{1}{1-\cos 2x} = \dfrac{1}{2}\csc^2 x$

$LHS = \dfrac{1}{1-\cos 2x} = \dfrac{1}{1-(1-2\sin^2 x)} = \dfrac{1}{2\sin^2 x}$

$= \dfrac{1}{2} \cdot \dfrac{1}{\sin^2 x} = \dfrac{1}{2}\csc^2 x = RHS$ ✓