6.2 #1-41 odd - due Mon. 01/09

6.1 #1-69 odd (proofs) - due Tues. 01/17

Due Mon. 01/23:

- 6.3 #1-24 all; 30-36 all; 49-93 odd Double & Half-angle Identities
- 6.5 #1-24 all ; #25-55 odd Inverse Trig Functions

Test #3 - Tues. 01/24 (6.1, 6.2, 6.3, 6.5)

Upcoming: Solving Trig Equations

- 6.6 #1-21 odd finding solutions between 0 and 2pi
  #61-69 odd finding all possible solutions (+2pi*k)
  #71-83 odd

Review

$= \cos\left(\frac{210^\circ}{2}\right)$

$\cos(105^\circ) = \cos(60^\circ + 45^\circ) = \cos 60 \cos 45 - \sin 60 \sin 45$

$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$

$= \frac{\sqrt{2} - \sqrt{6}}{4} =$

$\cos\frac{210}{2} = -\sqrt{\frac{1+\cos 210}{2}}$

$= -\sqrt{\frac{1 - \sqrt{3}/2}{2}} = -\sqrt{\left(\frac{2-\sqrt{3}}{2}\right) \cdot \frac{1}{2}}$

$= -\sqrt{\frac{2-\sqrt{3}}{4}} = -\frac{\sqrt{2-\sqrt{3}}}{2}$

$\tan\left(\frac{\pi}{8}\right) =$

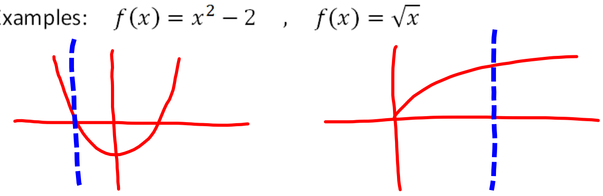
$\tan\frac{\pi/4}{2} = \frac{1 - \cos\pi/4}{\sin\pi/4} = \frac{1 - \sqrt{2}/2}{1/\sqrt{2}} = \left(1 - \frac{\sqrt{2}}{2}\right) \cdot \frac{\sqrt{2}}{1}$

$= \boxed{\sqrt{2} - 1}$

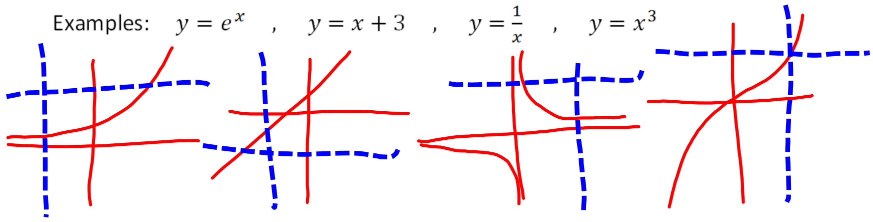
Inverse Trigonometric Functions

Recall from Algebra:

- f is a **function** if each input value (x) has a unique output $f(x)$.
  Examples: $f(x) = x^2 - 2$, $f(x) = \sqrt{x}$



- f is **one-to-one** if, in addition, each y corresponds to only one x.
  Examples: $y = e^x$, $y = x + 3$, $y = \frac{1}{x}$, $y = x^3$



- If f is a one-to-one function, we can define its inverse $f^{-1}(x)$.
  Note that this notation is not exponentiation, i.e. $f^{-1}(x) \neq \frac{1}{f(x)}$

$x^{-n} = \frac{1}{x^n}$

- $f(x)$ and $g(x)$ are **inverses** if
  $(f \circ g)(x) = f\big(g(x)\big) = x = g\big(f(x)\big) = (g \circ f)(x)$,
  that is, **inverse functions "undo" each other**.

Example: $f(x) = x^3$, $g(x) = \sqrt[3]{x}$

$(f \circ g)(x) = (\sqrt[3]{x})^3 = x$

$(g \circ f)(x) = \sqrt[3]{x^3} = x$

What do we mean by an Inverse Trig function?

Recall that **for a basic Trigonometric function**, e.g. $f(x) = \sin x$,

- **The input (x) is an angle**
- **The output $f(x)$ is a ratio of sides**

So **for an inverse Trigonometric function,**

- **The input (x) is a ratio of sides**
- **The output $f(x)$ is an angle**

Construction of the inverse of $f(x) = \sin x$:

$f(x) = x^3 - 8$

$y = x^3 - 8$

$x = y^3 - 8$

$x + 8 = y^3$

$\sqrt[3]{x+8} = y$

$f^{-1}(x) = \sqrt[3]{x+8}$

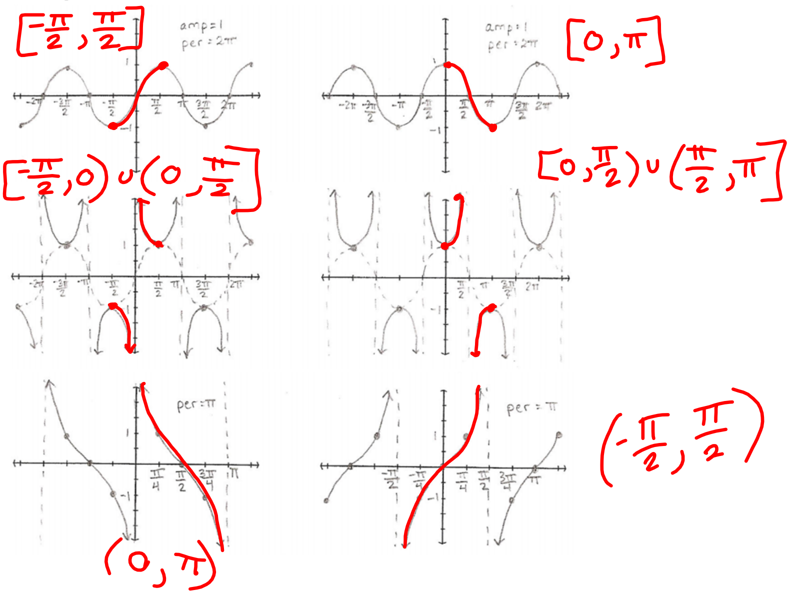
$y = \sin x$

$x = \sin y$

$y =$ the angle whose sine value is x

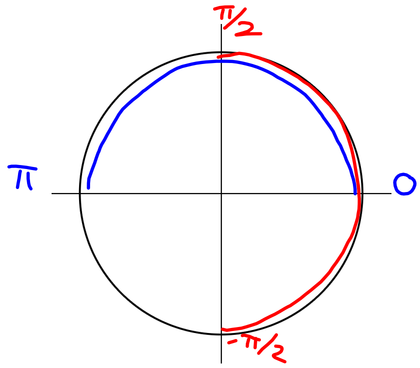
$f^{-1}(x) = \sin^{-1} x = \arcsin x$

But Trigonometric functions aren't one-to-one – how is the inverse defined? We must restrict the domain!

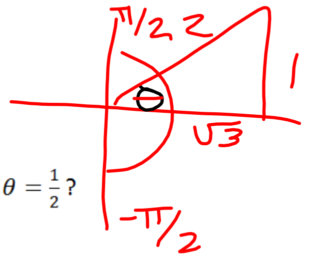


Summary of Restricted Domains:

Interval	Functions	Quadrants
$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$	$\sin x$, $\csc x$, $\tan x$	IV & I
$(0,\pi)$	$\cos x$, $\sec x$, $\cot x$	I & II



Evaluate the inverse trigonometric expression.

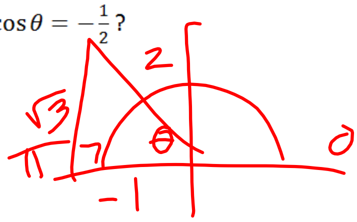


$\sin^{-1}\left(\frac{1}{2}\right) = \boxed{\frac{\pi}{6}}$

In words: What angle θ, between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ (the restricted domain for sine) is such that $\sin\theta = \frac{1}{2}$?

$\cos^{-1}\left(-\frac{1}{2}\right) = \boxed{\frac{2\pi}{3}}$

In words: What angle θ, between 0 and π (the restricted domain for cosine) is such that $\cos\theta = -\frac{1}{2}$?



$\tan^{-1}(-1) = \boxed{-\frac{\pi}{4}}$