

6.2 #1-41 odd - due Mon. 01/09

6.1 #1-69 odd (proofs) - due Tues. 01/17

Due Mon. 01/23:

- 6.3 #1-24 all; 30-36 all; 49-93 odd Double & Half-angle Identities
- 6.5 #1-24 all ; #25-55 odd Inverse Trig Functions

Test #3 - Tues. 01/24 (6.1, 6.2, 6.3, 6.5)

Upcoming: Solving Trig Equations

- 6.6 #1-21 odd finding solutions between 0 and 2π
#61-69 odd finding all possible solutions ($+2\pi \cdot k$)
#71-83 odd

$$1 + 4/2$$

$$(1 + 4)/2$$

$$\begin{aligned}
 1. \quad \text{LHS} &= \sin(a-b) - \sin(a+b) \\
 &= \sin a \cos b - \cos a \sin b - (\sin a \cos b + \cos a \sin b) = \\
 &= \sin a \cos b - \cos a \sin b - \sin a \cos b - \cos a \sin b = \\
 &= -2 \cos a \sin b = \text{RHS}
 \end{aligned}$$

$$2. \quad \frac{2 \cos 2x}{\sin 2x} = \cot x - \tan x$$

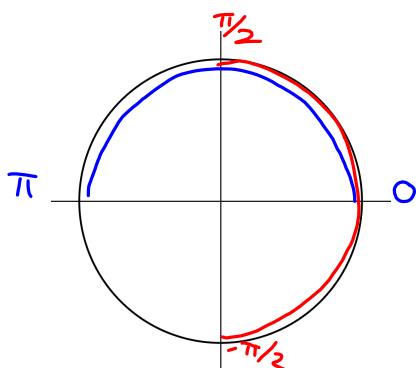
$$\text{LHS} = \frac{2(\cos^2 x - \sin^2 x)}{2 \sin x \cos x} = \frac{\cos^2 x}{\sin x \cos x} - \frac{\sin^2 x}{\sin x \cos x} =$$

$$= \frac{\cancel{\cos x} \cos x}{\cancel{\sin x} \cos x} - \frac{\cancel{\sin x} \sin x}{\cancel{\sin x} \cos x} = \cot x - \tan x = \text{RHS}$$

$$\begin{aligned}
 \text{RHS} &= \frac{\cos x}{\sin x} - \frac{\sin x}{\cos x} = \frac{\cos x \cdot \cos x}{\sin x \cos x} - \frac{\sin x \cdot \sin x}{\cos x \sin x} \\
 &= \frac{\cos^2 x}{\sin x \cos x} - \frac{\sin^2 x}{\sin x \cos x} = \frac{\cos^2 x - \sin^2 x}{\sin x \cos x} = \\
 &= \frac{2(\cos^2 x - \sin^2 x)}{2 \sin x \cos x} = \frac{2 \cos 2x}{\sin 2x} = \text{LHS} \quad \checkmark
 \end{aligned}$$

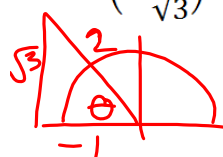
Summary of Restricted Domains:

Interval	Functions	Quadrants
$(-\frac{\pi}{2}, \frac{\pi}{2})$	$\sin x, \csc x, \tan x$	<u>IV & I</u>
$(0, \pi)$	$\cos x, \sec x, \cot x$	<u>I & II</u>

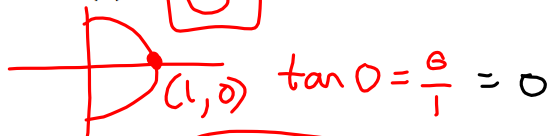


Evaluate.

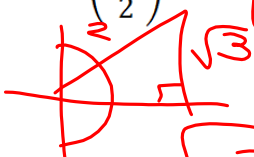
$$\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right) = \boxed{\frac{2\pi}{3}}$$



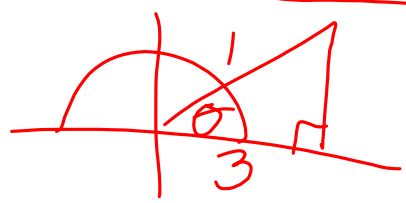
$$\tan^{-1}(0) = \boxed{0}$$



$$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \boxed{\frac{\pi}{3}}$$

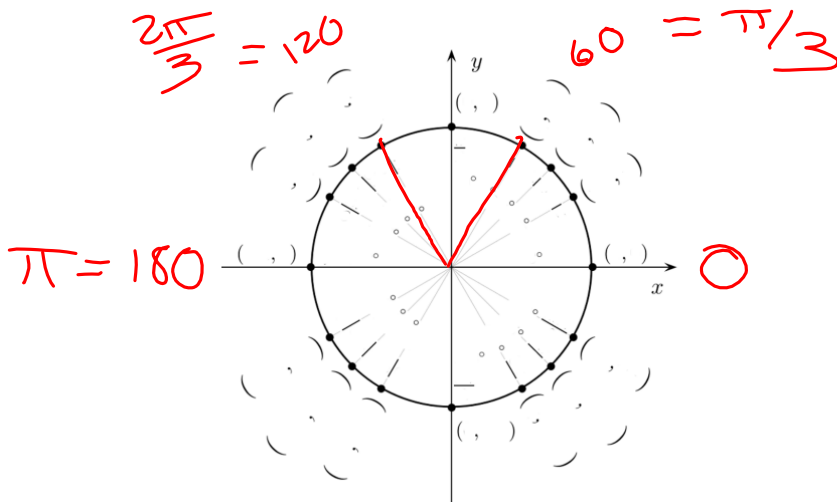


$$\cos^{-1}(3) = \text{undefined}$$



$$\csc^{-1}(-2) = \boxed{-\frac{\pi}{6}}$$





What happens when we compose a Trigonometric function with its inverse?

According to the definition,

$f(x)$ and $g(x)$ are inverses if $f(g(x)) = x$ and $g(f(x)) = x$
(for all x -values in the respective domains of g and f)

We would then expect

$$\sin(\sin^{-1} x) = x \text{ and } \sin^{-1}(\sin x) = x$$

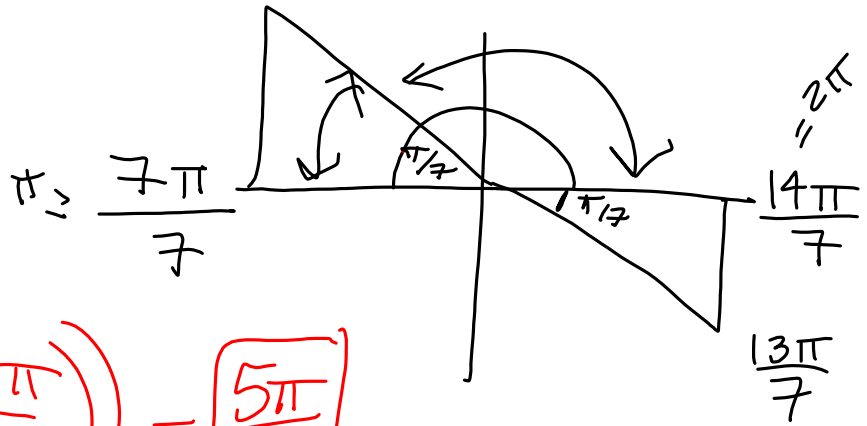
$$\sin\left(\sin^{-1}\frac{1}{2}\right) = \sin\frac{\pi}{6} = \frac{1}{2} \quad \sin^{-1}\left(\sin\left(\frac{5\pi}{6}\right)\right) = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$$

$$\sin^{-1}\left(\sin\left(-\frac{\pi}{6}\right)\right) = \sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6} \quad \cos^{-1}\left(\cos\left(\frac{8\pi}{7}\right)\right) = \frac{6\pi}{7}$$

$\frac{7\pi}{7} = \pi$

$$\sin(\sin^{-1} 3) = \text{undefined}$$

$$\cot^{-1}\left(\cot\frac{13\pi}{7}\right) = \boxed{\frac{6\pi}{7}}$$



$$\cot^{-1}\left(\cot\frac{5\pi}{7}\right) = \boxed{\frac{5\pi}{7}}$$

Evaluate:

$$\cos^{-1}\left(\cos\left(\frac{12\pi}{7}\right)\right) = \boxed{\frac{2\pi}{7}}$$

$$\tan^{-1}\left(\tan\left(\frac{4\pi}{5}\right)\right) = \boxed{\frac{-\pi}{5}}$$

$$\sec^{-1}\left(\sec\left(-\frac{4\pi}{5}\right)\right) = \boxed{\frac{4\pi}{5}}$$

