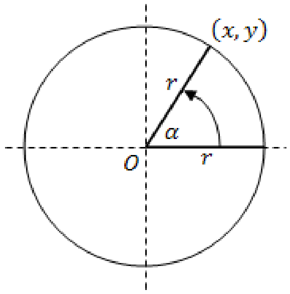
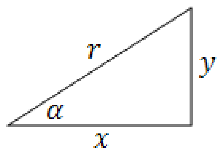
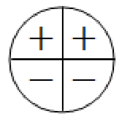
Trigonometry Graphing Guide

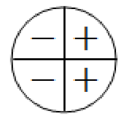
By: THE BEST GROUP (Megan, Faith, & Tommy ☺)

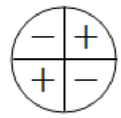


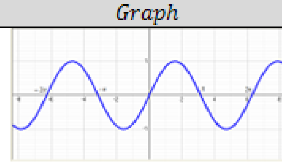
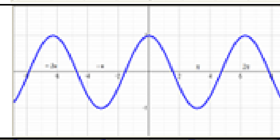
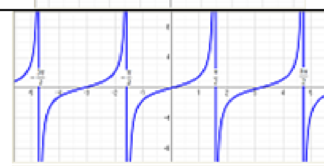
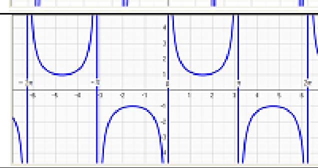
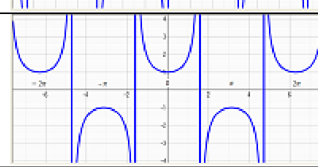
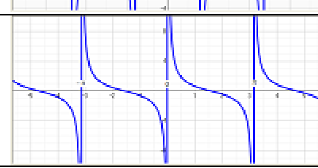


Signs of various trigonometric functions as a function of quadrant.

sin 

cos 

tan 

Function	*Definition*	*Periodic Function*	*Graph*
sin	$\sin\alpha = \frac{y}{r}$	$\sin\alpha = \sin(\alpha + 2\pi n)$	
cos	$\cos\alpha = \frac{x}{r}$	$\cos\alpha = \cos(\alpha + 2\pi n)$	
tan	$\tan\alpha = \frac{y}{x}$	$\tan\alpha = \tan(\alpha + \pi n)$	
csc	$\csc\alpha = \frac{r}{y} = \frac{1}{\sin\alpha}$	$\csc\alpha = \csc(\alpha + 2\pi n)$	
sec	$\sec\alpha = \frac{r}{x} = \frac{1}{\cos\alpha}$	$\sec\alpha = \sec(\alpha + 2\pi n)$	
cot	$\cot\alpha = \frac{x}{y} = \frac{1}{\tan\alpha}$	$\cot\alpha = \cot(\alpha + \pi n)$	

n – *any integer number*, $0, \pm 1, \pm 2$ ....

	Sine	Cosecant	Cosine	Secant	Tangent	Cotangent
Period	2 π	2 π	2 π	2 π	π	π
Asymptotes (if any)	n/a	Where sine is 0	n/a	Where cosine is 0	Half periods of π	intervals of π (including 0)

For a Trigonometric function in the form
y=af[b(x+c/b)]+d

Y=3sinx amplitude

Y=sin2x period

Y= sin(x+π/2) horz shift

Y= sinx +1 vert shift

Amplitude= |a| (a is always positive) *If there is a negative sign (-) in front of a, then you need to *flip* the graph

Period= original period of the function (π or 2 π)/ |b| *the original period of the graph is divided by the coefficient of the X

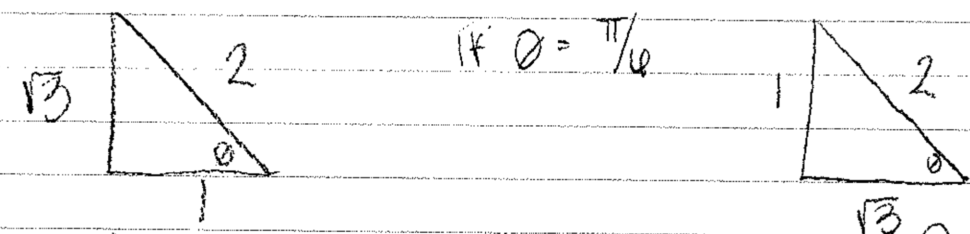
Horizontal shift – the number added or subtracted to the x (positive values move it to the left, negative values to the right) *if there is a period modifier (a *b*) then you also must divide the horizontal shift value by that value

Vertical shift – the number added or subtracted to the end of a function (positive numbers make it go up that amount, negative amounts make it go down by that amount) mandy nettles

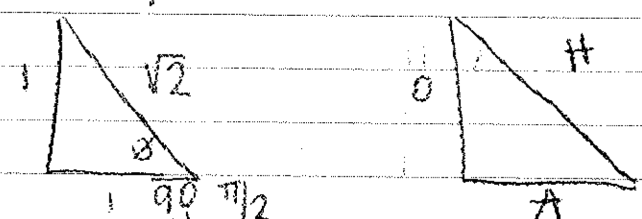
basic trig func / evaluate inverse fn's

- $\pi/3 = 60°$ $\pi/4 = 45°$ $2\pi = 0°$ or $360°$
- $\pi/6 = 30°$ $\pi/2 = 90°$

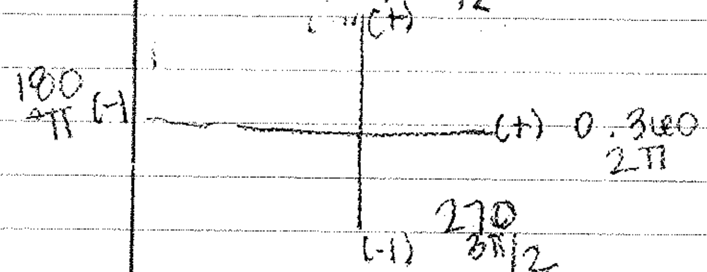
if $\theta = \pi/3$



if $\theta = \pi/4$



$\sin = o/h$ $\csc = h/o$
$\cos = a/h$ $\sec = h/a$
$\tan = o/a$ $\cot = a/o$

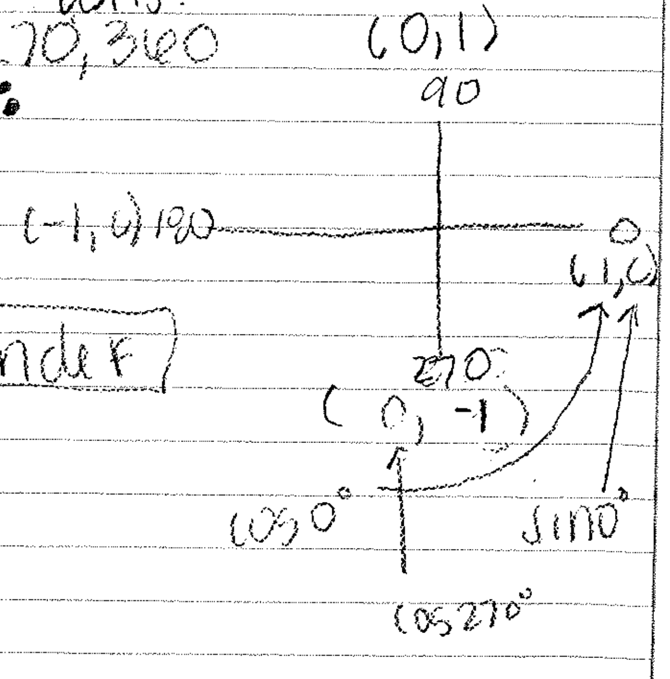


quadrantal angle: angle whose terminal side falls on an axis:
ex: 0, 90, 180, 270, 360

find the tan of 0°:

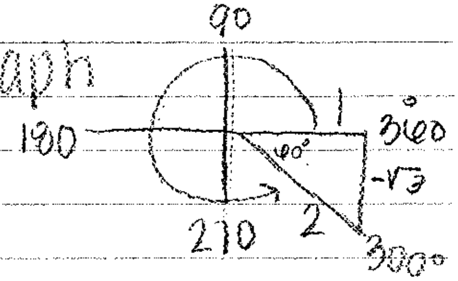
$\tan 0° = \frac{\sin 0°}{\cos 0°} = \frac{0}{1} = \boxed{0°}$

$\sec 270° = \frac{1}{\cos 270} = \frac{1}{0} = \boxed{\text{undef}}$

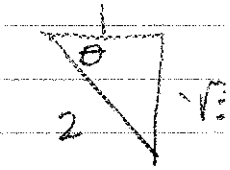


how to find the trig function of a given angle

ex: $\tan 300^\circ$



1. first find the angle on the graph
2. find the reference $\theta = 360 - 300$
   $= 60^\circ$
3. find side lengths based on ref θ
4. find the trig function of the angle.



$\tan 300^\circ = \frac{o}{a} = \frac{-\sqrt{3}}{1} = \boxed{-\sqrt{3}}$

how to convert to radians

degree to radians

$\frac{x^\circ}{1} \cdot \frac{\pi}{180^\circ}$

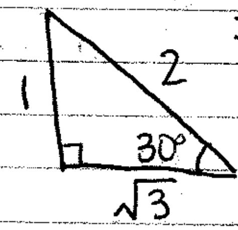
radians to degree

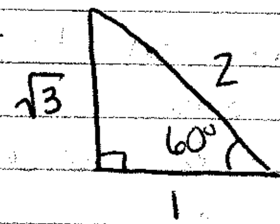
$x\pi \cdot \frac{180^\circ}{\pi}$

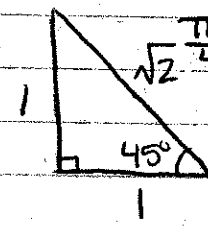
Déjà Monroe-Parker

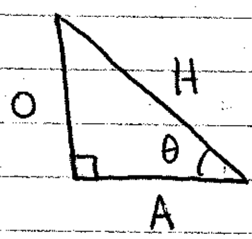
Basic Trigonometric Functions / Unit Circle

$\frac{\pi}{3} = 60^\circ$ reference angle
$\frac{\pi}{6} = 30^\circ$ reference angle
$\frac{\pi}{4} = 45^\circ$ reference angle

$\frac{\pi}{6}$ 

$\frac{\pi}{3}$ 

$\frac{\pi}{4}$ 



H - hypotenuse
O - opposite
A - adjacent

$\sin\theta = \frac{O}{H}$ $\cos\theta = \frac{A}{H}$ $\tan\theta = \frac{O}{A} = \frac{\sin\theta}{\cos\theta}$

$\csc\theta = \frac{1}{\sin\theta} = \frac{H}{O}$ $\sec\theta = \frac{1}{\cos\theta} = \frac{H}{A}$

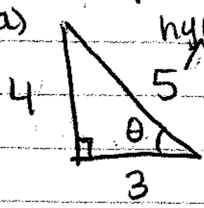
$\cot\theta = \frac{1}{\tan\theta} = \frac{A}{O} = \frac{\cos\theta}{\sin\theta}$

Examples:

a)

O
opposite the angle

hypotenuse H



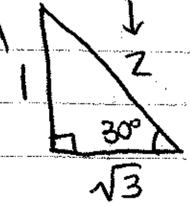
adjacent to the angle
A

$\sin\theta = \frac{O}{H} = \frac{4}{5}$
$\cos\theta = \frac{A}{H} = \frac{3}{5}$
$\tan\theta = \frac{O}{A} = \frac{4}{3}$
$\csc\theta = \frac{H}{O} = \frac{5}{4}$
$\sec\theta = \frac{H}{A} = \frac{5}{3}$
$\cot\theta = \frac{A}{O} = \frac{3}{4}$

b)

opposite the angle

hypotenuse



adjacent to the angle

$\sin\theta = \frac{O}{H} = \frac{1}{2}$
$\cos\theta = \frac{A}{H} = \frac{\sqrt{3}}{2}$
$\tan\theta = \frac{O}{A} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
$\csc\theta = \frac{H}{O} = \frac{2}{1} = 2$
$\sec\theta = \frac{H}{A} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$
$\cot\theta = \frac{A}{O} = \frac{\sqrt{3}}{1} = \sqrt{3}$

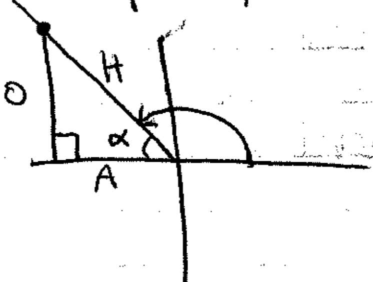
~~Refer to Section 5.3~~

Refer to Section 5.3 - Trigonometric Functions of Any Angle for Next Portion Below

The reference angle is the acute angle (between 0° and 90°) between the terminal side of the angle and the ~~x-axis~~ x-axis.

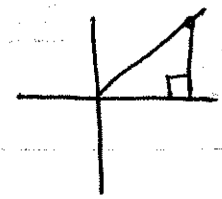
Symbols to implicate reference angle:

α alpha β beta ω omega θ theta



α = some angle

#° unknown

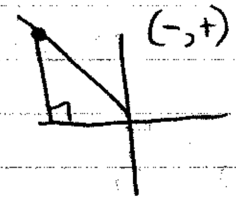


(+,+) I

ALL are positive (all)

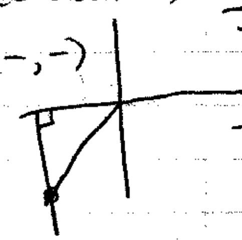
*H = hypotenuse is always positive

*O & A = are legs of the right triangle and can be positive or negative depending on the quadrant they are located in



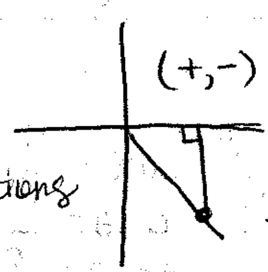
(−,+) II

SINE is positive (Students)



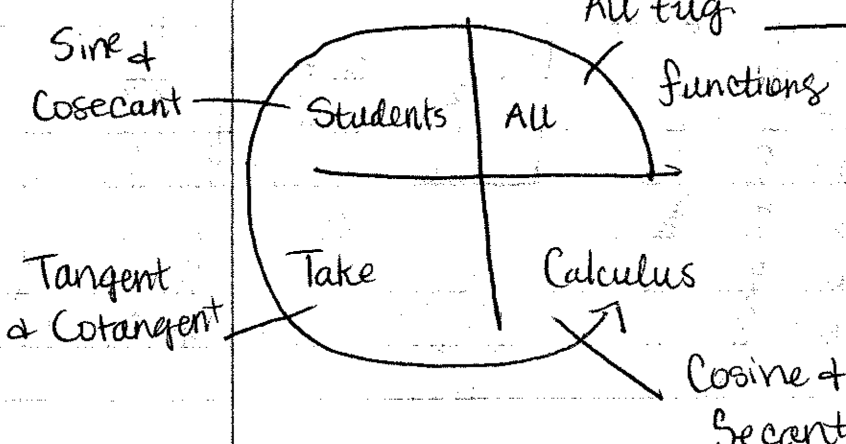
(−,−) III

TAN is positive (Take



(+,−) IV

COSINE is positive (Calculus)



Law of Sines

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ ⟵ Formula used for AAS triangles or ASS triangles

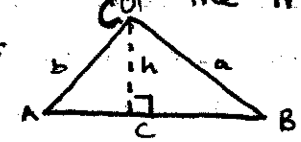
Derivative

$\sin B = \frac{h}{a}$ $\qquad$ $\sin A = \frac{h}{b}$ $\qquad$ * h = the length of the altitude of the triangle

$h = a\sin B$ $\qquad$ $h = b\sin A$

$$a\sin B = b\sin A$$

*



$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$\frac{c}{\sin C} = \frac{b}{\sin B}$ & $\frac{c}{\sin C} = \frac{a}{\sin A}$

Example of AAS triangle

Solve ΔABC if $A = 42°$, $B = 63°$, and $c = 18$

1. $A + B + C = 180°$

$42° + 63° + C = 180°$

$C = 75°$

2. $\frac{a}{\sin A} = \frac{c}{\sin C}$

$\frac{a}{\sin 42°} = \frac{18}{\sin 75°}$

$a = \frac{18 \sin 42°}{\sin 75°} \approx 12$

3. $\frac{b}{\sin B} = \frac{c}{\sin C}$

$\frac{b}{\sin 63°} = \frac{18}{\sin 75°}$

$b = \frac{18 \sin 63°}{\sin 75°} \approx 17$

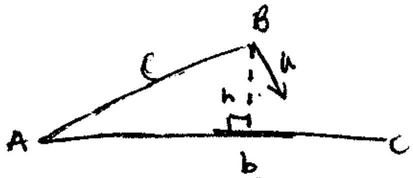
Solution is $C = 75°$, $a \approx 12$, and $b \approx 17$.

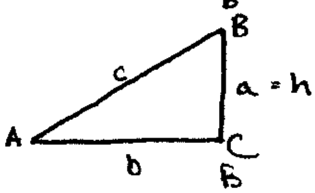
ASS, The Problematic Triangle

There may be 0, 1, or 2 solutions:

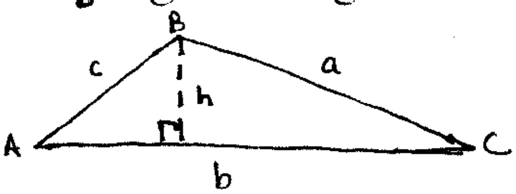
Case 1 – $\angle A$ is an acute angle.

1. $a < h$; there is no possible triangle.
2. $a = h$; there is one triangle, a right triangle.
3. $h < a < c$; there are two possible triangles. One has all acute angles, and the other has one obtuse angle.
4. $a \geq c$; there is one triangle, which is not a right triangle.

1. 

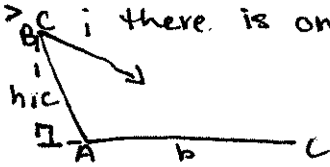
2. 

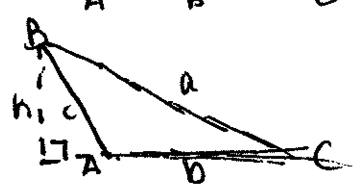
3. 

4. 

Case 2 – $\angle A$ is an obtuse angle.

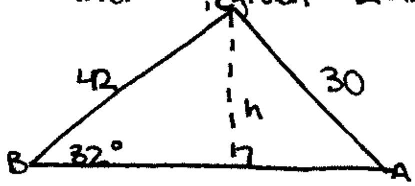
1. $a \leq c$; there is no triangle.
2. $a > c$; there is one triangle.

1. 

2. 

Example of ASS Triangle

a. Find A, given $\triangle ABC$ with $B = 32°$, $a = 42$, and $b = 30$.



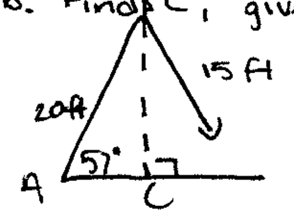
$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

$$\frac{\sin A}{42} = \frac{\sin 32°}{30}$$

$$A = \sin^{-1}\left(\frac{42 \sin 32°}{30}\right) \approx 0.7419$$

$$A \approx 48° \text{ or } 132°$$

b. Find C, given $\triangle ABC$ with $A = 57°$, $a = 15$ ft, and $c = 20$ ft



$$\frac{\sin C}{20 \text{ ft}} = \frac{\sin 57}{15 \text{ ft}}$$

$$C = \sin^{-1}\left(\frac{20 \sin 57°}{15}\right) \approx \sin^{-1}(1.1182) = \text{no solution}$$

For more problems, go to p. 541

Law of Cosines

$a^2 = b^2 + c^2 - 2bc(\cos A)$

$b^2 = a^2 + c^2 - 2ac(\cos B)$ ← Formula used for SAS

$c^2 = a^2 + b^2 - 2ab(\cos C)$ or SSS triangle

Derivative

$c = \sqrt{(a\cos C - b)^2 + (a\sin C - 0)^2}$

$c^2 = a^2\cos^2 C - 2ab\cos C + b^2 + a^2\sin^2 C$

$c^2 = a^2(\cos^2 C + \sin^2 C) + b^2 - 2ab\cos C$

$c^2 = a^2 + b^2 - 2ab\cos C$

Triangle Inequality

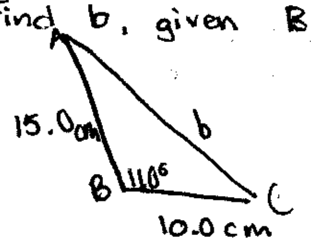
$a < b + c$

$b < a + c$ ← for all triangles

$c < a + b$

Example of SAS triangle

Find b, given $B = 110.0°$, $a = 10.0$ centimeters, and $c = 15.0$ centimeters.



$b^2 = a^2 + c^2 - 2ac(\cos B)$

$b^2 = (10)^2 + (15)^2 - 2(10)(15)\cos 110°$

$b = \sqrt{(10)^2 + (15)^2 - 2(10)(15)\cos 110°}$

$b \approx 20.7$ centimeters

Example of SSS Triangle

Find B, given $a = 32$ ft, $b = 20$ ft, and $c = 40$ ft

$b^2 = a^2 + c^2 - 2ac(\cos B)$

$\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$

$\cos B = \dfrac{(32)^2 + (40)^2 - (20)^2}{2(32)(40)}$

$B = \cos^{-1}\left(\dfrac{(32)^2 + (40)^2 - (20)^2}{2(32)(40)}\right)$

$B \approx 30°$

Area of a Triangle

$A = \frac{1}{2}bh$ ← used only for SAS triangles

$h = bc\sin A \,/\, ab\sin C \,/\, ac\sin B$

$K = \frac{1}{2}\,bc\sin A$

$K = \frac{1}{2}\,ab\sin C$

$K = \frac{1}{2}\,ac\sin B$

Example of Finding the Area of an SAS Triangle

Given $A = 62°$, $b = 12$ meters, and $c = 5$ meters, find the area.

$K = \frac{1}{2}bc\sin A = \frac{1}{2}(12)(5)(\sin 62°) \approx 26\ m^2$

Heron's Formula

$K = \sqrt{s(s-a)(s-b)(s-c)}$ ← used only for SSS triangles

$s = \frac{1}{2}(a+b+c)$

Example of Finding the Area of an SSS Triangle

Given $a = 7$ meters, $b = 15$ meters, and $c = 12$ meters, find the area.

$s = \dfrac{a+b+c}{2} = 17$

$K = \sqrt{17(17-7)(17-15)(17-12)}$

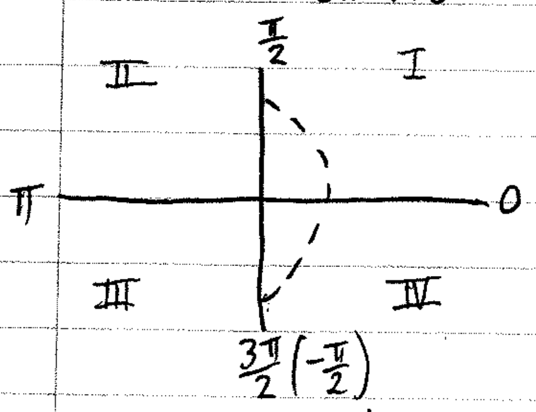
$K = \sqrt{1700} \approx 41$ meters^2

For more problems, go to p. 550 - 551

Molly Broyles

Evaluating Inverse Functions

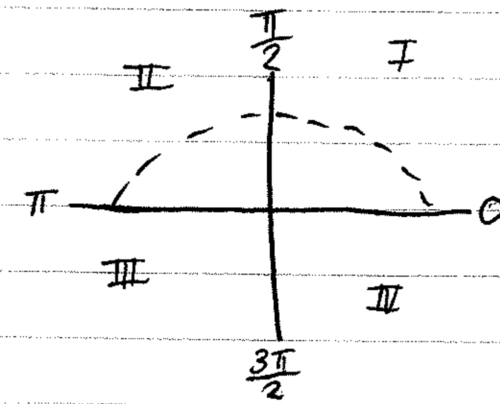
Restricted Domains:



sin, csc, tan

Inverse must be between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$

If $\sin^{-1}$, $\csc^{-1}$, or $\tan^{-1}$ is in the third quadrant, the angle must not be written in a form which exceeds $\frac{\pi}{2}$. For example: if the angle is $\frac{11\pi}{6}$, it must be written as $-\frac{\pi}{6}$.



cos, sec, cot

Inverse must be between 0 and π

If a function is multiplied by the inverse of that function, and that angle is in the restricted domain, then the function and the inverse cancel out. Thus, the solution is the original angle.

$\sin^{-1}\left(\sin\frac{\pi}{6}\right)=\frac{\pi}{6}$

$\sin^{-1}\left(\sin\frac{7\pi}{6}\right)\neq\frac{7\pi}{6}$

$\csc^{-1}\left(\sin\frac{\pi}{6}\right)\neq\frac{\pi}{6}$

Evaluating Trig. Identities

I) Identities that Involve $(\alpha \pm \beta)$

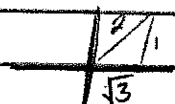
· Identities to Know: Sum & Difference Identities

- Find the exact value of the expression.

Pg 484

① $\sin(45° + 30°)$

$= \sin 45 \cos 30 + \cos 45 \sin 30$



$\sin 45 = \frac{1}{\sqrt{2}}$ $\cos 45 = \frac{1}{\sqrt{2}}$

$\sin 30 = 1/2$ $\cos 30 = \sqrt{3}/2$

$(1/\sqrt{2})(\sqrt{3}/2) + (1/\sqrt{2})(\frac{1}{2})$

$\sqrt{3}/2\sqrt{2} + \frac{1}{2\sqrt{2}}$

$= \boxed{\frac{1 + \sqrt{3}}{2\sqrt{2}}}$

Step 1: Check that you wrote down the formula correctly.
Step 2: Expand the Identity.
Step 3: Solve for the Δ.
Step 4: Multiply
Step 5: Check for common denominator
Step 6: Add

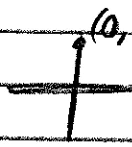
⑬ $\cos 212° \cos 122° + \sin 212° \sin 122°$

$= \cos(212° - 122°)$

$= \cos 90°$

$= \boxed{0}$

$\begin{array}{r} 212 \\ -122 \\ \hline 90° \end{array}$



* (212° & 122° are not going to create perfect Δ.)

- Write in terms of a single trig. function.

Pg 484

⑲ $\sin 7x \cos 2x - \cos 7x \sin 2x$

$= \sin(7x - 2x)$

$= \boxed{\sin 5x}$

Step 1: Did you copy correctly?
Step 2: What's the identity?
Step 3: Simplify

(29) $\dfrac{\tan 3x + \tan 4x}{1 - \tan 3x \tan 4x} \rightarrow \tan(3x + 4x) \rightarrow \tan 7x$

-Find the exact value of the given functions.

Pg 484

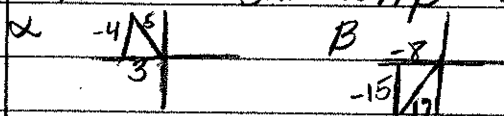
(31) Given $\tan\alpha = -4/3$, α in Q II, and $\tan\beta = 15/8$, β in Q III, find

a) $\sin(\alpha - \beta)$ b) $\cos(\alpha + \beta)$ c) $\tan(\alpha - \beta)$

Step 1: Solve for the Δ using the given.
Step 2: Expand the function.
Step 3: Find the sin + cos for each.
Step 4: Multiply
Step 5: Check for common denominators
Step 6: Add

a) $\sin(\alpha - \beta)$

$\tan\alpha = -4/3 \; \frac{O}{A}$ $\tan\beta = 15/8 \; \frac{O}{A}$



$\sin\alpha\cos\beta - \cos\alpha\sin\beta$

$(-4/5)(-8/17) - (3/5)(-15/17)$

$(32/85) - (-45/85)$

$= \boxed{\dfrac{77}{85}}$

b) $\cos(\alpha + \beta)$

$\cos\alpha\cos\beta - \sin\alpha\sin\beta$

$(3/5)(-8/17) - (-4/5)(-15/17)$

$(24/85) - (60/85)$

$= \boxed{\dfrac{-36}{85}}$

Can You Do It?

c) $\tan(\alpha - \beta) \rightarrow \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

II) Double + Half-Angle Identities

- Write in terms of a single trig. function

Pg 491

① $2\sin 2\alpha \cos 2\alpha$

$= \sin 2\alpha \cos 2\alpha + \sin 2\alpha \cos 2\alpha$

$= \sin(2\alpha + 2\alpha)$

$= \boxed{\sin 4\alpha}$

Step 1: Did you copy the problem correctly?
Step 2: Expand
Step 3: Substitute Trig. Identities

③ $1 - 2\sin^2 5\beta$

$= \cos^2 5\beta - \sin^2 5\beta$

$= \cos 2(5\beta)$

$= \cos 10\beta$

- Use the half-angle identities to find the exact value.

⑨ $\sin 75^\circ \times 2 \rightarrow \sin \frac{150}{2}$

$= \pm\sqrt{\dfrac{1-\cos(150)}{2}}$

$= \pm\sqrt{\dfrac{1-(-\sqrt{3}/2)}{2}}$

$= \pm\dfrac{\sqrt{\frac{2}{2}+\sqrt{3}/2}}{\sqrt{2}} \rightarrow \dfrac{\sqrt{\frac{2+\sqrt{3}}{2}}}{\sqrt{2}}$

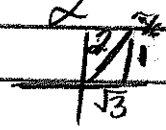
$= \pm\dfrac{\sqrt{2+\sqrt{3}}}{\sqrt{2}} \cdot \dfrac{\sqrt{2}}{1} \rightarrow \dfrac{2\sqrt{6}}{\sqrt{2}}$

$\boxed{= \pm 2\sqrt{3}}$

Step 1: Double the angle + put it over 2.
Step 2: Apply Identity
Step 3: Solve the Δ
Step 4: Seperate + Solve
Step 5: Simplify + Rationalize
Step 6: Don't Forget the $\pm$

(23) $\cos \pi/12 \times 2 \rightarrow \cos \dfrac{\pi/6}{2}$

$= \pm \sqrt{\dfrac{1+\cos(\pi/6)}{2}}$

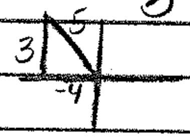


$= \pm \dfrac{\sqrt{2/2 + \sqrt{3}/2}}{\sqrt{2}} \rightarrow \dfrac{\sqrt{2+\sqrt{3}}}{\sqrt{2}} \cdot \dfrac{1}{\sqrt{2}} \rightarrow \boxed{\dfrac{\pm\sqrt{2+\sqrt{3}}}{2}}$

- Find the exact value of $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$ given the following information

Pg 491-2

(25) $\cos\theta = \dfrac{-4}{5}\,\dfrac{A}{H}$, θ is in Q II



Step 1: Solve Δ to get the angle.
Step 2: Expand Identity
Step 3: Apply Solved Δ to Equation
Step 4: Multiply

a) $\sin 2\theta \rightarrow 2\sin\theta\cos\theta$

$\dfrac{2}{1}\left(\dfrac{3}{5}\right)\left(\dfrac{-4}{5}\right)$

$= \boxed{\dfrac{-24}{25}}$

b) $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

$\dfrac{2}{1}\left(\dfrac{3}{-4}\right) \div 1 - \left(\dfrac{3}{-4}\right)\left(\dfrac{3}{-4}\right)$

$\dfrac{3}{-2} \div \dfrac{16}{16} - \left(\dfrac{9}{16}\right)$

$\dfrac{3}{-2} \cdot \dfrac{16}{7} \rightarrow \dfrac{48}{-14} \rightarrow \boxed{\dfrac{-24}{7}}$

c) $\cos 2\theta$

Can You Do It?

Team Sauce Drip Drippin Saucy Sauce

By: Sir Saucy, Honey Mustard, Madam Dripper-Sauce, and Lil Drip with the Skip

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Arc Length

Angular Speed

Linear Speed

$$v = \frac{s}{t}$$

Angular Speed

$$\omega = \frac{\theta}{t}$$

ω "omega" Ω

Arc Length

$$s = r\theta$$

$$\boxed{v = r\omega}$$

Relating Linear & Angular Speed

$$v = \frac{s}{t} = \frac{r\theta}{t} = r \cdot \frac{\theta}{t} = r\omega$$

$\boldsymbol{r}$ = radius or distance from the center of rotation $(in, cm, km, etc.)$

$\boldsymbol{s}$ = arc length or linear distance along the circumference of a circle $(in, cm, km, etc.)$

$\boldsymbol{\theta}$ = angle or amount of rotation $(deg, rad, revolutions, etc.)$

$\boldsymbol{t}$ = time $(sec, min, hours, years, etc.)$

$\boldsymbol{v} = \frac{\text{linear distance}}{\text{time}} =$ linear speed $\left(\frac{km}{s}, \frac{mi}{h}, etc.\right)$

$\boldsymbol{\omega} = \frac{\text{amount of rotation}}{\text{time}} =$ angular speed $\left(\frac{rev}{min}, \frac{deg}{s}, etc.\right)$

Proper steps to solve:

1. Identify given (sort variables)
2. Determine which equation to use
3. Rearrange equation to solve for variable
4. Plug in quantities (check for units)
5. Use dimensional analysis to get to proper units

Examples

Look on *brewermath.com November 7, 2016*

Angular Speed (Section 5.1)

1. A wheel is rotating at 50 revolutions per minute. Find the angular speed in radians per second.

2. Find the angular speed, in radians per second hand on a clock.
3. The turntable of a record player turns at 33 (1/3) revolutions per minute. Find the angular speed in radians per second.

Linear Speed (Section 5.1)

1. Each car tire has a radius of 15 inches. The tires are rotating at 450 rev per minute. Find the speed of the automobile to the nearest mph.
2. Wind machine is used to generate electricity. The machine has propeller blades that are 12 ft. in length. If the propeller is rotating at 3 rev per second what is the linear speed in ft per second of the tips of the blades?
3. A wheel with a 15 inch diameter rotates at a rate of 6 radians per second. What is the linear speed of a point on its rim in feet per minute?

Distance and Angular Speed

A merry-go-round horse is 11.6 meters from the center. the merry go round makes 14 1/4 rev per ride in 5 min.

a. How many meters to the nearest meter does the horse travel?
b. How fast is it moving in meters per second?

Arc Length

What is the length of arc S?