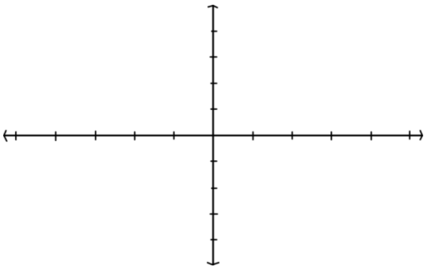
$y = -2\sec 17x$

amplitude:

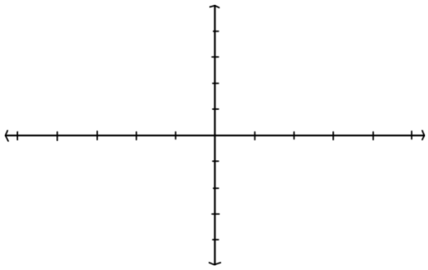
period:



$y = \frac{\pi}{2}\tan\left(\frac{3}{\pi}x\right)$

amplitude:

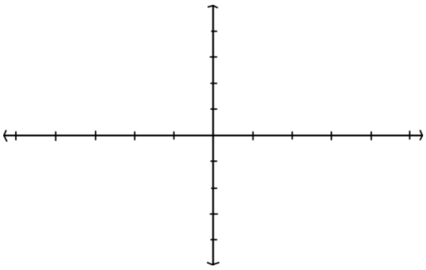
period:



$y = -\frac{2}{3}\sin\left(\frac{5}{\sqrt{2}}x\right)$

amplitude:

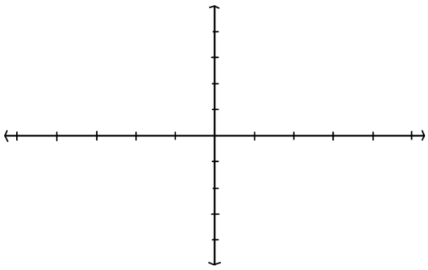
period:



$y = -\frac{2}{3}\csc\left(\frac{5}{\sqrt{2}}x\right)$

amplitude:

period:



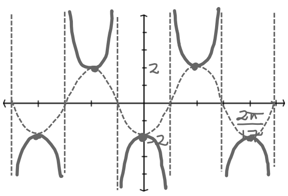
$y = -2\sec 17x$

amplitude:

2

period:

$\frac{2\pi}{17}$



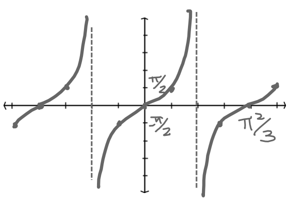
$y = \frac{\pi}{2}\tan\left(\frac{3}{\pi}x\right)$

amplitude: "

$\pi/2$

period:

$\frac{\pi}{3/\pi} = \frac{\pi^2}{3}$



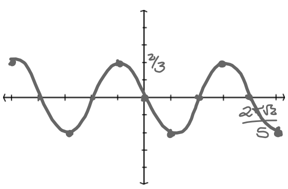
$y = -\frac{2}{3}\sin\left(\frac{5}{\sqrt{2}}x\right)$

amplitude:

$2/3$

period:

$\frac{2\pi}{5/\sqrt{2}} = \frac{2\pi\sqrt{2}}{5}$



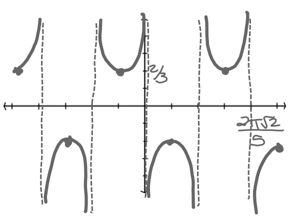
$y = -\frac{2}{3}\csc\left(\frac{5}{\sqrt{2}}x\right)$

amplitude:

$2/3$

period:

$\frac{2\pi\sqrt{2}}{5}$



Goal: Transform a trigonometric function of the form $y = f(x)$ to one of the form $y = af(bx + c) + d$ by observing changes in amplitude and period, as well as horizontal and vertical shifts.

Recall:

- Constants that are multiplied (divided) result in a stretching/scaling of the graph (amplitude/period changes), that we show by changing the scale on our axes
- Constants that are added (subtracted) result in shifting of the graph
- Constants outside the function (a & d) affect it vertically, as we would expect
- Constants inside the function (b & c) affect it horizontally, opposite of what we would expect

$y = af(bx)$ ✓ scaling

$y = f(x+c)+d$ shifting

$y = f(x+c)+d$ shifting

outside - vertically, as we would expect

inside - horizontally, opposite

d=vertical shift

d>0 up

d<0 down

c=horizontal shift ("phase shift" = -c)

c>0 left

c<0 right

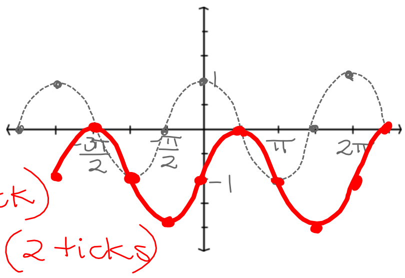
$y = \cos(x - \frac{\pi}{2}) - 1$

amplitude:
1

period:
2π

horiz. shift:
$\frac{\pi}{2}$ right (1 tick)

vert. shift:
1 down (2 ticks)



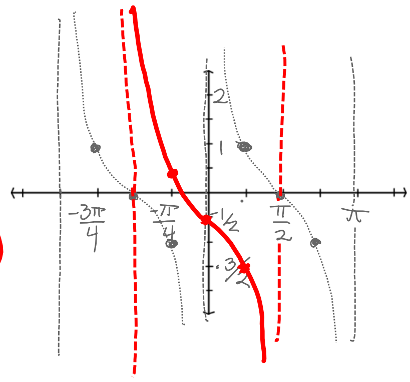
$y = \cot(x + \frac{\pi}{2}) - \frac{1}{2}$

"amplitude:"
1

period:
π

horiz. shift:
$\frac{\pi}{2}$ left (2 ticks)

vert. shift:
$\frac{1}{2}$ down (1 tick)



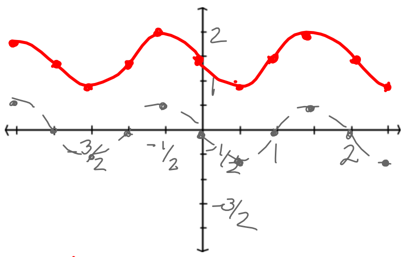
$y = -\frac{1}{2}\sin \pi x + \frac{3}{2}$

amplitude:
$\frac{1}{2}$

period:
$\frac{2\pi}{\pi} = 2$

horiz. shift:
N/A
0

vert. shift:
$\frac{3}{2}$ up (3 ticks)

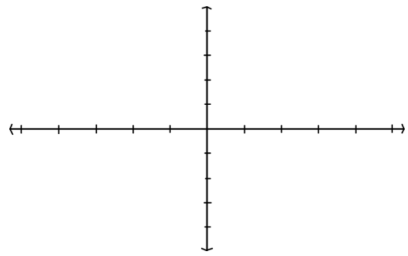


amplitude:

period:

horiz. shift:

vert. shift:



Graphing Trigonometric Functions continued...

Goal: Transform a trigonometric function of the form $y = f(x)$ to one of the form $y = af(bx + c) + d$ by observing changes in amplitude and period, as well as horizontal and vertical shifts.

Recall:

- Constants that are multiplied (divided) result in a stretching/scaling of the graph (amplitude/period changes), that we show by changing the scale on our axes
- Constants that are added (subtracted) result in shifting of the graph
- Constants outside the function (a & d) affect it vertically, as we would expect
- Constants inside the function (b & c) affect it horizontally, opposite of what we would expect

Note:

When both b and c are present (i.e. when b is anything other than 1), the horizontal shift is not just $c = \frac{c}{1}$, as it is affected by the presence of b. In this case (and in general), the horizontal shift is $\frac{c}{b}$, which we can more easily see by factoring b out in the general equation: $\boxed{y = af\left[b\left(x + \frac{c}{b}\right)\right] + d}$

Summary:

For a Trigonometric function of the form $y = af\left[b\left(x+\frac{c}{b}\right)\right]+d$,

Amplitude $= |a|$ (note that amplitude is always positive)

Period $= \dfrac{original\ period\ of\ the\ function\ (\pi\ or\ 2\pi)}{|b|}$

Horizontal shift $= \dfrac{c}{b}$, $left\ if\ \frac{c}{b}>0$, $right\ if\ \frac{c}{b}<0$

Vertical shift $= d$, $up\ if\ d>0$, $down\ if\ d<0$

$y=-2\cos\left(\frac{\pi}{3}x-\frac{3\pi}{2}\right)+1$

amplitude: $=-2\cos\left[\pi/3\left(x-\frac{9}{2}\right)\right]+1$

2

period:

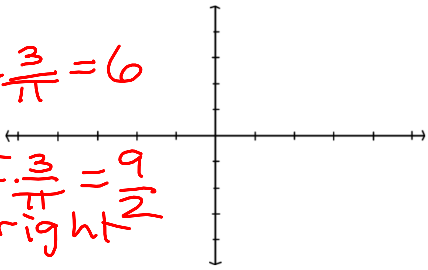
$\frac{2\pi}{\pi/3}=\frac{2\pi}{1}\cdot\frac{3}{\pi}=6$

horiz. shift:

$\frac{3\pi/2}{\pi/3}=\frac{3\pi}{2}\cdot\frac{3}{\pi}=\frac{9}{2}$ right

vert. shift:

1 up



$y=2\sec\left(\frac{\pi}{2}x-\pi\right)$

amplitude:

2

period:

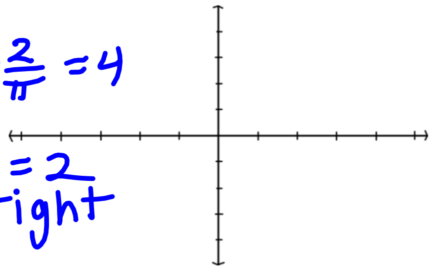
$\frac{2\pi}{\pi/2}=2\pi\cdot\frac{2}{\pi}=4$

horiz. shift:

$\frac{\pi}{\pi/2}=\pi\cdot\frac{2}{\pi}=2$ right

vert. shift:

N/A



$y=-\frac{1}{3}\tan\left(\frac{1}{4}x+\frac{\pi}{4}\right)-\frac{1}{3}$

amplitude:

1/3

period:

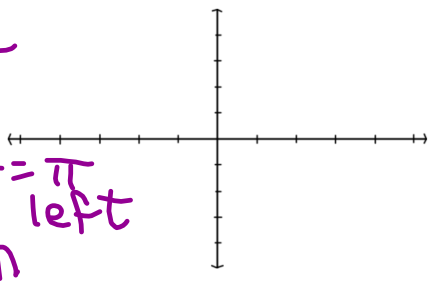
$\frac{\pi}{1/4}=4\pi$

horiz. shift:

$\frac{\pi/4}{1/4}=\frac{\pi}{4}\cdot\frac{4}{1}=\pi$ left

vert. shift:

1/3 down



amplitude:

period:

horiz. shift:

vert. shift: