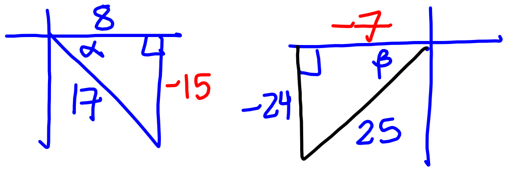
Given $\cos\alpha = \frac{8}{17}$, $\alpha \in QIV$

$\sin\beta = \frac{-24}{25}$, $\beta \in QIII$

find $\sin(\alpha+\beta)$, $\cos(\alpha+\beta)$, $\tan(\alpha+\beta)$, & determine the quadrant in which $\alpha+\beta$ lies.



*Pythagorean triples that are useful to know:
3,4,5 ; 5,12,13 ; 7,24,25 ;
& 8,15,17

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta = \left(\frac{-15}{17}\right)\left(\frac{-7}{25}\right) + \left(\frac{8}{17}\right)\left(\frac{-24}{25}\right) = \boxed{\frac{-87}{425}}$$

105 − 192

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta = \left(\frac{8}{17}\right)\left(\frac{-7}{25}\right) - \left(\frac{-15}{17}\right)\left(\frac{-24}{25}\right) = \boxed{\frac{-416}{425}}$$

−56 − 360

$$\tan(\alpha+\beta) = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = \boxed{\frac{87}{416}}$$

$\alpha+\beta \in \boxed{QII}$

$$\boxed{\cos 2\theta = \cos^2\theta - \sin^2\theta}$$

$$\left[\begin{array}{l} \sin^2\theta + \cos^2\theta = 1 \\ \sin^2\theta = 1 - \cos^2\theta \\ \cos^2\theta = 1 - \sin^2\theta \end{array}\right]$$

$$\cos 2\theta = \cos^2\theta - (1 - \cos^2\theta)$$

$$\boxed{\cos 2\theta = 2\cos^2\theta - 1}$$

$$\cos 2\theta = (1 - \sin^2\theta) - \sin^2\theta$$

$$\boxed{\cos 2\theta = 1 - 2\sin^2\theta}$$

$$\tan 2\theta = \tan(\theta+\theta) = \frac{\tan\theta+\tan\theta}{1-\tan\theta\tan\theta}$$

$$\tan(a+b) = \frac{\tan a+\tan b}{1-\tan a\tan b}$$

$$\boxed{\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}}$$

Given $\sin\theta = -\frac{2}{3}$, $\theta \in QIII$,

Find $\sin 2\theta$, $\cos 2\theta$, $\tan 2\theta$, and the quadrant in which 2θ lies.

Double-Angle Identities

$\sin 2x = 2\sin x\cos x$

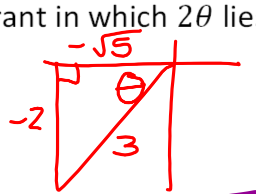
$\cos 2x = \cos^2 x - \sin^2 x$

$= 2\cos^2 x - 1$

$= 1 - 2\sin^2 x$

$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$

$$\sin 2\theta = 2\sin\theta\cos\theta = \frac{2}{1}\left(\frac{-2}{3}\right)\left(\frac{-\sqrt{5}}{3}\right)$$



$$\boxed{\sin 2\theta = \frac{4\sqrt{5}}{9}}$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = \left(\frac{-\sqrt{5}}{3}\right)^2 - \left(\frac{-2}{3}\right)^2 = \frac{5}{9} - \frac{4}{9} = \boxed{\frac{1}{9} = \cos 2\theta}$$

$$\boxed{\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta}} \boxed{= 4\sqrt{5}}$$

$$\boxed{2\theta \in QI}$$

Half-Angle Identities

$$\sin\frac{x}{2} = ?$$

$$\cos 2\theta = 1 - 2\sin^2\theta$$

Let $\theta = \frac{x}{2}$.

$$\cos 2\left(\frac{x}{2}\right) = 1 - 2\sin^2\left(\frac{x}{2}\right)$$

$$\cos x = 1 - 2\left(\sin\frac{x}{2}\right)^2$$

$$2\left(\sin\frac{x}{2}\right)^2 = 1 - \cos x$$

$$\left(\sin\frac{x}{2}\right)^2 = \frac{1-\cos x}{2}$$

$$\boxed{\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}}$$

$x^2 = 4$ $f^2(x) = c$

$x = \pm 2$ $f(x) = \pm\sqrt{c}$

$$\cos\frac{x}{2} = ?$$

$$\cos 2\theta = 2\cos^2\theta - 1$$

$$\cos \cancel{2}\left(\frac{x}{\cancel{2}}\right) = 2\left(\cos\left(\frac{x}{2}\right)\right)^2 - 1$$

$$\frac{1+\cos x}{2} = \frac{\cancel{2}\left(\cos\frac{x}{2}\right)^2}{\cancel{2}}$$

$$\pm\sqrt{\frac{1+\cos x}{2}} = \cos\frac{x}{2}$$

Half-Angle Identities

$$\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}} \quad , \quad \cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$$

$$\tan\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{1+\cos x}}$$

$$= \frac{\sin x}{1+\cos x}$$

$$\boxed{\tan\frac{x}{2} = \frac{1-\cos x}{\sin x}}$$

$$\tan\frac{7\pi}{12} = \tan\frac{7\pi/6}{2}$$

$$\frac{7\pi}{12} = \frac{x}{2}$$

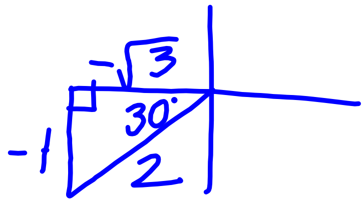
$$\cancel{2}\cdot\frac{7\pi}{\cancel{12}_{6}} = x$$

$$\boxed{\frac{7\pi}{6} = x}$$

$$\tan\frac{x}{2} = \frac{1-\cos x}{\sin x}$$

$$\tan\frac{7\pi}{12} = \frac{1-\cos\frac{7\pi}{6}}{\sin\frac{7\pi}{6}}$$

$$= \frac{1-\left(\frac{-\sqrt{3}}{2}\right)}{\frac{-1}{2}}$$



$$= \left(1+\frac{\sqrt{3}}{2}\right)\cdot\frac{-2}{1} = \boxed{-2-\sqrt{3}}$$