Review

cos (105°) =

$$\cos(45^\circ+60^\circ) = \cos 45^\circ \cos 60^\circ - \sin 45^\circ \sin 60^\circ$$

$$= \frac{\sqrt{2}}{2}\cdot\frac{1}{2} - \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} = \boxed{\frac{\sqrt{2}-\sqrt{6}}{4}}$$

OR

$$\cos\frac{210^\circ}{2} = -\sqrt{\frac{1+\cos 210^\circ}{2}} = -\sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}\cdot\frac{2}{2}} = \boxed{\frac{-\sqrt{2-\sqrt{3}}}{2}}$$

equal!

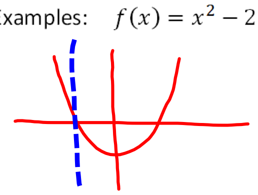
$$\tan\left(\frac{\pi}{8}\right) = \tan\frac{\pi/4}{2} = \frac{1-\cos\pi/4}{\sin\pi/4} = \frac{1-\sqrt{2}/2}{\frac{1}{\sqrt{2}}} = \left(1-\frac{\sqrt{2}}{2}\right)\cdot\frac{\sqrt{2}}{1} =$$

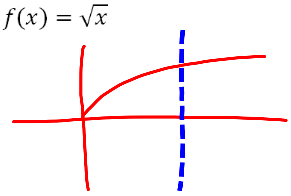
$$= 1\cdot\frac{\sqrt{2}}{1} - \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{1} = \sqrt{2} - \frac{2}{2} = \boxed{\sqrt{2}-1}$$

<u>**Inverse Trigonometric Functions**</u>

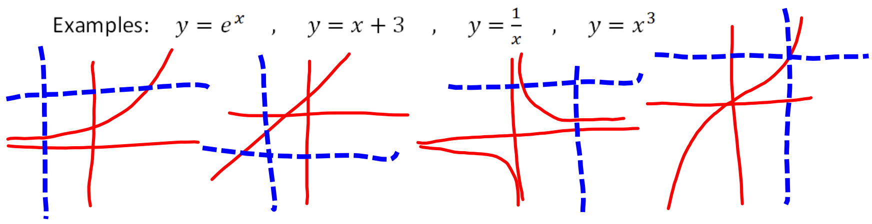
Recall from Algebra:

- f is a **function** if each input value (x) has a unique output $f(x)$.
  Examples: $f(x) = x^2 - 2$, $f(x) = \sqrt{x}$





- f is **one-to-one** if, in addition, each y corresponds to only one x.
  Examples: $y = e^x$, $y = x + 3$, $y = \frac{1}{x}$, $y = x^3$



- If f is a one-to-one function, we can define its inverse $f^{-1}(x)$.
  Note that this notation is not exponentiation, i.e. $f^{-1}(x) \neq \frac{1}{f(x)}$

$X^{-n} = \frac{1}{X^n}$

- $f(x)$ and $g(x)$ are **inverses** if
  $(f \circ g)(x) = f(g(x)) = x = g(f(x)) = (g \circ f)(x)$,
  that is, **inverse functions "undo" each other**.

Example: $f(x) = x^3$, $g(x) = \sqrt[3]{x}$

$(f \circ g)(x) = (\sqrt[3]{x})^3 = x$

$(g \circ f)(x) = \sqrt[3]{(x^3)} = x$

What do we mean by an Inverse Trig function?

Recall that **for a basic Trigonometric function**, e.g. $f(x) = \sin x$,

- **The input (x) is an angle**
- **The output $f(x)$ is a ratio of sides**

So **for an inverse Trigonometric function,**

- **The input (x) is a ratio of sides**
- **The output $f(x)$ is an angle**

Construction of the inverse of $f(x) = \sin x$:

$f(x) = x^3 - 8$

$y = x^3 - 8$

$x = y^3 - 8$

$x + 8 = y^3$

$\sqrt[3]{(x+8)} = \sqrt[3]{y^3}$

$\sqrt[3]{x+8} = y$

$\boxed{f^{-1}(x) = \sqrt[3]{x+8}}$

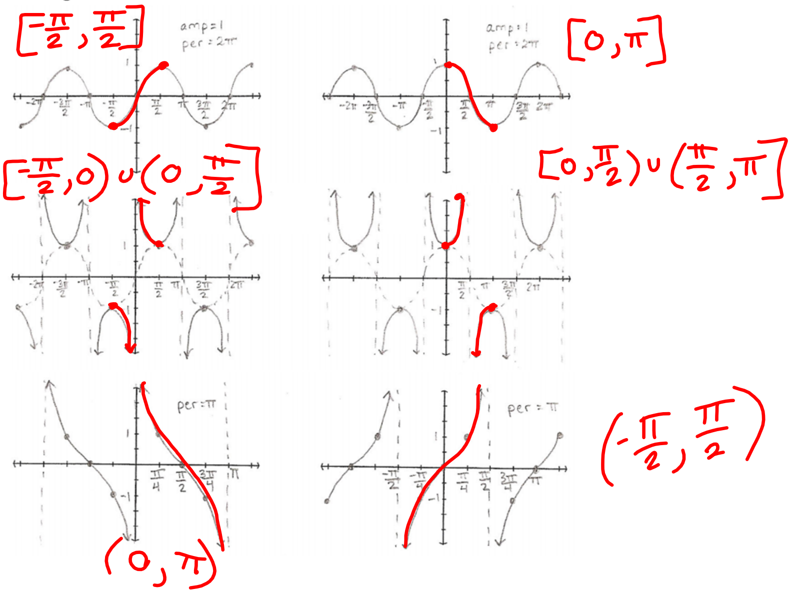
$y = \sin x$

$x = \sin y$

$y =$ the angle whose sine value is x

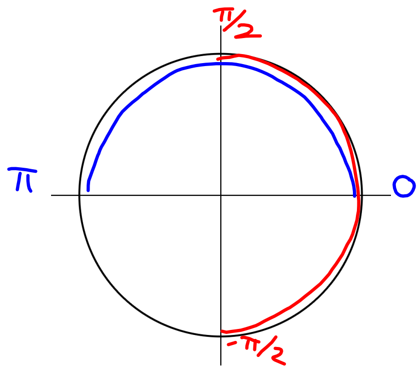
$f^{-1}(x) = \sin^{-1}(x)$
or
$= \arcsin x$

But Trigonometric functions aren't one-to-one – how is the inverse defined? We must restrict the domain!



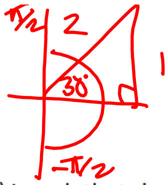
Summary of Restricted Domains:

Interval	Functions	Quadrants
$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$	$\sin x, \csc x, \tan x$	IV & I
$(0,\pi)$	$\cos x\ , \sec x\ , \cot x$	I & II



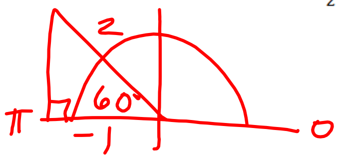
Evaluate the inverse trigonometric expression.

$\sin^{-1}\left(\frac{1}{2}\right) = \boxed{\frac{\pi}{6}}$



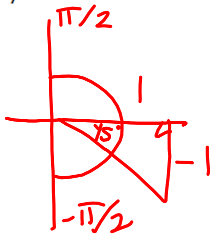
In words: What angle θ, between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ (the restricted domain for sine) is such that $\sin\theta = \frac{1}{2}$?

$\cos^{-1}\left(-\frac{1}{2}\right) = \boxed{\frac{2\pi}{3}}$



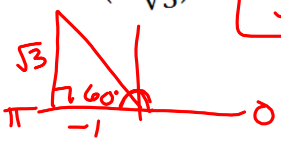
In words: What angle θ, between 0 and π (the restricted domain for cosine) is such that $\cos\theta = -\frac{1}{2}$?

$\tan^{-1}(-1) = \boxed{-\frac{\pi}{4}}$

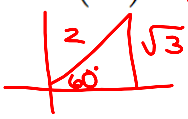


Evaluate.

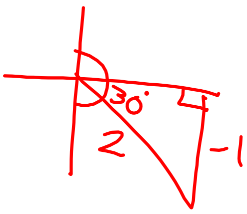
1. $\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right) = \boxed{\frac{2\pi}{3}}$



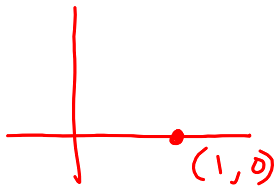
2. $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \boxed{\frac{\pi}{3}}$



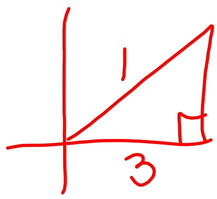
3. $\csc^{-1}(-2) = \csc^{-1}\left(\frac{2}{-1}\right) = \boxed{-\frac{\pi}{6}}$



4. $\tan^{-1}(0) = \tan^{-1}\left(\frac{0}{1}\right) = \boxed{0}$



5. $\cos^{-1}(3) = \cos^{-1}\left(\frac{3}{1}\right) = \boxed{\text{undefined}}$



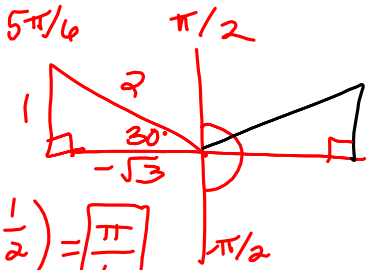
What happens when we compose a Trigonometric function with its inverse?

According to the definition,

$f(x)$ and $g(x)$ are inverses if $f\big(g(x)\big) = x$ and $g\big(f(x)\big) = x$
(for all x-values in the respective domains of g and f)

We would then expect

$\sin(\sin^{-1} x) = x$ and $\sin^{-1}(\sin x) = x$

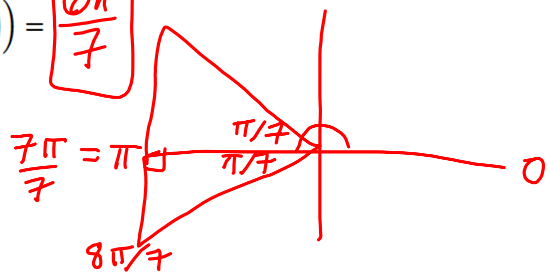


$\sin\left(\sin^{-1}\frac{1}{2}\right) = \sin\frac{\pi}{6} = \boxed{\frac{1}{2}}$

$\sin^{-1}\left(\sin\left(\frac{5\pi}{6}\right)\right) = \sin^{-1}\left(\frac{1}{2}\right) = \boxed{\frac{\pi}{6}}$

$\sin^{-1}\left(\sin\left(-\frac{\pi}{6}\right)\right) = \sin^{-1}\left(-\frac{1}{2}\right) = -\pi/6$

$\cos^{-1}\left(\cos\left(\frac{8\pi}{7}\right)\right) = \boxed{\frac{6\pi}{7}}$



$\sin(\sin^{-1} 3) =$ undefined