Solve for $x \in [0, 2\pi)$

20. $\tan^2 x + \tan x - \sqrt{3} = \sqrt{3}\tan x$

$$\underbrace{\tan^2 x + \tan x}\ \underbrace{- \sqrt{3}\tan x - \sqrt{3}} = 0$$

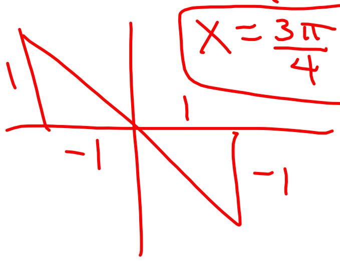
$$\tan x(\tan x + 1) - \sqrt{3}(\tan x + 1) = 0$$

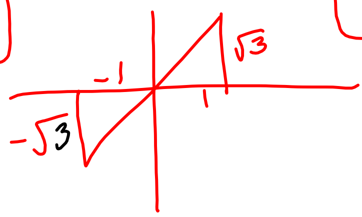
$$(\tan x + 1)(\tan x - \sqrt{3}) = 0$$

$$\tan x + 1 = 0 \qquad \tan x - \sqrt{3} = 0$$

$$\tan x = -1 \qquad \tan x = \sqrt{3}$$

$\boxed{x = \frac{3\pi}{4}, \frac{7\pi}{4}}$ $\boxed{x = \frac{\pi}{3}, \frac{4\pi}{3}}$





$x \in [0, 2\pi)$

22. $\cos^4 x = \cos^2 x$

$$\cos^4 x - \cos^2 x = 0$$

$$\cos^2 x(\cos^2 x - 1) = 0$$

$$\cos^2 x = 0 \qquad \cos^2 x = 1$$

$$\cos x = 0 \qquad \cos x = \pm 1$$

$\boxed{x = \frac{\pi}{2}, \frac{3\pi}{2}}$ $\boxed{x = 0, \pi}$

New Directions: Find ALL the solutions (not just in $[0,2\pi)$)

62. $\sec 3x - \frac{2\sqrt{3}}{3} = 0$

$\sec\theta = \frac{2}{\sqrt{3}}$

$\theta = \frac{\pi}{6}, \frac{11\pi}{6}$

$\theta + 2\pi k$, $k \in \mathbb{Z}$

$= \theta$ & every integer multiple of 2π more or less than it

$\sec 3x = \frac{2\sqrt{3}}{3}$

$\sec \boxed{3x} = \frac{2}{\sqrt{3}}$

$\frac{3x}{3} = \frac{\frac{\pi}{6} + 2\pi k}{3}$; $\frac{3x}{3} = \frac{\frac{11\pi}{6} + 2\pi k}{3}$

$\boxed{x = \frac{\pi}{18} + \frac{2\pi k}{3} \; ; \; x = \frac{11\pi}{18} + \frac{2\pi k}{3}}$

all the x's

68. $\cos\left(2x - \frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2} = -\frac{1}{\sqrt{2}}$

$2x - \frac{\pi}{4} = \frac{3\pi}{4} + 2\pi k$; $2x - \frac{\pi}{4} = \frac{5\pi}{4} + 2\pi k$

$2x = \pi + 2\pi k$

$2x = \frac{3\pi}{2} + 2\pi k$

$\boxed{x = \frac{\pi}{2} + \pi k}$

$\boxed{x = \frac{3\pi}{4} + \pi k}$

Solve for $x \in [0, 2\pi)$.

$(\sin x - \cos x)^2 = 1^2$

* caution: squaring both sides may introduce extraneous solutions!

$\sin^2 x - 2\sin x\cos x + \cos^2 x = 1$

$(\sin^2 x + \cos^2 x) - 2\sin x \cos x = 1$

$1 - \sin 2x = 1$

$0 = \sin 2x$

$0 \le x < 2\pi$

$0 \le 2x < 4\pi$

$2x = 0, \pi, 2\pi, 3\pi$

$x = \cancel{0}, \boxed{\pi/2}, \boxed{\pi}, \cancel{3\pi/2}$

$\sin 0 - \cos 0$

$0 - 1 = -1 \ne 1$

$\sin\frac{\pi}{2} - \cos\frac{\pi}{2}$

$1 - 0 = 1$

$\sin\pi - \cos\pi$

$0 - (-1) = 1$

$\sin\frac{3\pi}{2} - \cos\frac{3\pi}{2}$

$-1 - 0 = -1$

$\cos(4x) = \frac{1}{\sqrt{2}}$

$x \in [0, 2\pi)$

$0 \le x < 2\pi$

$0 \le 4x < 8\pi$

$\left(+2\pi \cdot \frac{4}{4} = \frac{8\pi}{4}\right)$

$4x = \frac{\pi}{4}, \frac{7\pi}{4}; \frac{9\pi}{4}, \frac{15\pi}{4}; \frac{17\pi}{4}, \frac{23\pi}{4}; \frac{25\pi}{4}, \frac{31\pi}{4}$

$\boxed{x = \frac{\pi}{16}, \frac{7\pi}{16}, \frac{9\pi}{16}, \frac{15\pi}{16}, \frac{17\pi}{16}, \frac{23\pi}{16}, \frac{25\pi}{16}, \frac{31\pi}{16}}$

$\tan(5x) = 0$

$0 \le x < 2\pi$

$0 \le 5x < 10\pi$

$5x = 0, \pi; 2\pi, 3\pi; 4\pi, 5\pi; 6\pi, 7\pi; 8\pi, 9\pi$

$$\boxed{x = 0, \frac{\pi}{5}, \frac{2\pi}{5}, \frac{3\pi}{5}, \frac{4\pi}{5}, \pi, \frac{6\pi}{5}, \frac{7\pi}{5}, \frac{8\pi}{5}, \frac{9\pi}{5}}$$

72. $\cos 2x = 2\cos x - 1$

$x \in [0, 2\pi)$

$\underbrace{\cos 2x} - 2\cos x + 1 = 0$

$2\cos^2 x - \cancel{1} - 2\cos x + \cancel{1} = 0$

$2\cos^2 x - 2\cos x = 0$

$2\cos x(\cos x - 1) = 0$

$2\cos x = 0, \quad \cos x - 1 = 0$

$\cos x = 0 \qquad \cos x = 1$

$$\boxed{x = \pi/2, 3\pi/2 \qquad x = 0}$$

$x \in [0, 2\pi)$

74. $\sin 4x - \cos 2x = 0$

$\sin 2(2x) - \cos 2x = 0$

$\boxed{\sin 2\theta = 2\sin\theta\cos\theta}$

$2\sin 2x \cos 2x - \cos 2x = 0$

$\cos 2x\,(2\sin 2x - 1) = 0$

$\cos 2x = 0$

$2x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$

$\boxed{x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}}$

$2\sin 2x - 1 = 0$

$2\sin 2x = 1$

$\sin 2x = 1/2$

$2x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$

$\boxed{x = \pi/12, 5\pi/12, 13\pi/12, 17\pi/12}$