78. $\cos 2x \cos x - \sin 2x \sin x = 0$

$\cos(2x + x) = 0$

$\cos 3x = 0$

$3x = \frac{\pi}{2}, \frac{3\pi}{2}; \frac{5\pi}{2}, \frac{7\pi}{2}; \frac{9\pi}{2}, \frac{11\pi}{2}$

$\boxed{x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}}$

$\boxed{\begin{array}{c} 0 \le x < 2\pi \\ 0 \le 3x < 6\pi \end{array}}$

82. $\cos 3x + \cos x = 0$

$\cos(2x + x) + \cos x = 0$

$\cos 2x \cos x - \sin 2x \sin x + \cos x = 0$

$(\cos^2 x - \sin^2 x)\cos x - (2\sin x \cos x)\sin x + \cos x = 0$

$\cos^3 x - \cos x \sin^2 x - 2\cos x \sin^2 x + \cos x = 0$

$\cos^3 x - 3\cos x \sin^2 x + \cos x = 0$

$\cos^3 x - 3\cos x(1 - \cos^2 x) + \cos x = 0$

$\cos^3 x - 3\cos x + 3\cos^3 x + \cos x = 0$

$4\cos^3 x - 2\cos x = 0$

$2\cos x(2\cos^2 x - 1) = 0$

$2\cos x = 0$ $\qquad$ $2\cos^2 x - 1 = 0$

$\cos x = 0$ $\qquad$ $\cos^2 x = 1/2$

$\boxed{x = \frac{\pi}{2}, \frac{3\pi}{2}}$ $\qquad$ $\cos x = \pm 1/\sqrt{2}$

$\boxed{x = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4}$

84. $2\sin x\cos x - 2\sqrt{2}\sin x - \sqrt{3}\cos x + \sqrt{6} = 0$

$2\sin x(\cos x - \sqrt{2}) - \sqrt{3}(\cos x - \sqrt{2}) = 0$

$(\cos x - \sqrt{2})(2\sin x - \sqrt{3}) = 0$

$\cos x = \sqrt{2}$ $\sin x = \sqrt{3}/2$

$x = \varnothing$ $\boxed{x = \pi/3, 2\pi/3}$

empty set (no solutions)

76. $\tan\frac{x}{2} = 1 - \cos x$

$\frac{1-\cos x}{\sin x} = 1 - \cos x$

$(1-\cos x) = \sin x(1-\cos x)$

$1(1-\cos x) - \sin(1-\cos x) = 0$

$(1-\cos x)(1-\sin x) = 0$

$1 = \cos x$ $1 = \sin x$

$\boxed{x = 0}$ $\boxed{x = \pi/2}$

7. Find all solutions (in radians) in the interval $0 \le x < 2\pi$. $\sin^2 x - \frac{1}{4} = 0$

$\sin^2 x = 1/4$

$\sin x = \pm \frac{1}{2}$

$\boxed{x = \pi/6, 5\pi/6, 7\pi/6, 11\pi/6}$

8. Find all solutions (in radians) in the interval $0 \le x < 2\pi$. $2\sin^3 x = \sin x$

$2\sin^3 x - \sin x = 0$

$\sin x(2\sin^2 x - 1) = 0$

$\sin x = 0$

$\boxed{x = 0, \pi}$

$\sin^2 x = 1/2$

$\sin x = \pm 1/\sqrt{2}$

$\boxed{x = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4}$

Bonus (10 points): Find all solutions (in radians) in the interval $0 \le x < 2\pi$.

$$\sin 3x + \sin x - \sin 2x = 0$$

$\sin(2x+x) + \sin x - 2\sin x\cos x = 0$

$\sin 2x\cos x + \cos 2x\sin x + \sin x - 2\sin x\cos x = 0$

$(2\sin x\cos x)\cos x + (\cos^2 x - \sin^2 x)\sin x + \sin x - 2\sin x\cos x = 0$

$2\sin x\cos^2 x + \sin x\cos^2 x - \sin^3 x + \sin x - 2\sin x\cos x = 0$

$3\sin x\cos^2 x - \sin^3 x + \sin x - 2\sin x\cos x = 0$

$3\sin x(1-\sin^2 x) - \sin^3 x + \sin x - 2\sin x\cos x = 0$

$3\sin x - 3\sin^3 x - \sin^3 x + \sin x - 2\sin x\cos x = 0$

$4\sin x - 4\sin^3 x - 2\sin x\cos x = 0$

$2\sin x\,(2 - 2\sin^2 x - \cos x) = 0$

$2\sin x = 0$

$\sin x = 0$

$\boxed{x = 0, \pi}$

$2 - 2(1-\cos^2 x) - \cos x = 0$

$\cancel{2-2} + \cos^2 x - \cos x = 0$

$\cos x(\cos x - 1) = 0$

$\cos x = 0$, $\cos x = 1$

$\boxed{x = \pi/2, 3\pi/2}$, 0

4 problems from solving equations handout;

7.5 #27-40