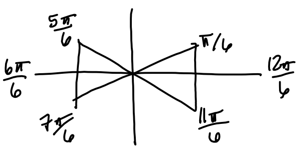
$$\sqrt{3}\cos x - \sin x = 1$$
$$(\sqrt{3}\cos x)^2 = (1 + \sin x)^2$$
$$3\cos^2 x = 1 + 2\sin x + \sin^2 x$$
$$3(1-\sin^2 x) = 1 + 2\sin x + \sin^2 x$$
$$3 - 3\sin^2 x = 1 + 2\sin x + \sin^2 x$$
$$0 = 4\sin^2 x + 2\sin x - 2$$
$$0 = 2\sin^2 x + \sin x - 1$$
$$0 = (2\sin x - 1)(\sin x + 1)$$
$$\sin x = \tfrac{1}{2} \qquad \sin x = -1$$
$$x = \frac{\pi}{6}, \cancel{\frac{5\pi}{6}}; \quad x = \frac{3\pi}{2}$$
$$\sqrt{3}\cos x - \sin x \stackrel{?}{=} 1$$
$$\frac{\pi}{6}: \sqrt{3}\cdot\frac{\sqrt{3}}{2} - \frac{1}{2} = 1 \checkmark$$
$$\frac{5\pi}{6}: \sqrt{3}\left(-\frac{\sqrt{3}}{2}\right) - \frac{1}{2} = -2$$
$$\frac{3\pi}{2}: \sqrt{3}(0) - (-1) = 1 \checkmark$$

$$\cot x = \tan(2x - 3\pi)$$
$$\cot x = \frac{\tan 2x - \cancel{\tan 3\pi}^{\,0}}{1 + \tan 2x \cancel{\tan 3\pi}^{\,0}}$$
$$\cot x = \tan 2x$$
$$\frac{1}{\tan x} = \frac{2\tan x}{1-\tan^2 x}$$

$$\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$$
$$\cot x = \frac{1}{\tan x}$$
$$\cot x = \tan\left(\tfrac{\pi}{2} - x\right)$$
$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

$$\frac{(\cancel{\tan x})(1-\tan^2 x)}{1}\cdot\frac{1}{\cancel{\tan x}} = \frac{2\tan x}{\cancel{1-\tan^2 x}}\cdot\frac{(\tan x)(\cancel{1-\tan^2 x})}{1}$$
$$1 - \tan^2 x = 2\tan x \cdot \tan x$$
$$1 - \tan^2 x = 2\tan^2 x$$
$$1 = 3\tan^2 x$$
$$\frac{1}{3} = \tan^2 x$$
$$\pm\frac{1}{\sqrt{3}} = \tan x$$



$$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}}$$
$$2x = \pi/3, 5\pi/3, 7\pi/3, 11\pi/3$$
$$2x - 3\pi = -\frac{8\pi}{3}, -\frac{4\pi}{3}, -\frac{2\pi}{3}, \frac{2\pi}{3}$$
$$\tan(2x-3\pi): \sqrt{3}\ \ -\sqrt{3}\ \ \sqrt{3}\ \ -\sqrt{3}$$
$$\cot x: \sqrt{3}\ \ \sqrt{3}\ \ \sqrt{3}\ \ -\sqrt{3}$$

checks all work ✓

$\sin 4x - 2\sin 2x = 0$

$0 \le x < 2\pi$

$0 \le 2x < 4\pi$

$\sin 2(2x) - 2\sin 2x = 0$

$2\sin 2x\cos 2x - 2\sin 2x = 0$

$2\sin 2x(\cos 2x - 1) = 0$

$2\sin 2x = 0$ $\quad$ $\cos 2x - 1 = 0$

$\sin 2x = 0$ $\quad$ $\cos 2x = 1$

$2x = 0, \pi, 2\pi, 3\pi$ $\quad$ $2x = 0, 2\pi$

$\boxed{x = 0, \pi/2, \pi, 3\pi/2}$ $\quad$ $x = 0, \pi$

$2\cos x + 2\sin x = \sqrt{6}$

$2(\cos x + \sin x) = \sqrt{6}$

$(\cos x + \sin x)^2 = \left(\frac{\sqrt{6}}{2}\right)^2$

$\cos^2 x + 2\sin x\cos x + \sin^2 x = \frac{6}{4}$

$(\sin^2 x + \cos^2 x) + 2\sin x\cos x = 3/2$

$1 + \sin 2x = \frac{3}{2}$

$\sin 2x = \frac{1}{2}$

$2x = \pi/6, 5\pi/6, 13\pi/6, 17\pi/6$

$x = \pi/12, 5\pi/12, \cancel{13\pi/12}, \cancel{17\pi/12}$

check for extraneous solutions

$$\underbrace{2\sec x\tan x + 2\sec x} + \underbrace{\tan x + 1} = 0$$

$$2\sec x(\tan x+1) + 1(\tan x+1) = 0$$

$$(\tan x+1)(2\sec x+1) = 0$$

$$\tan x = -1 \quad , \quad \cancel{\sec x = -1/2}$$

$$\boxed{x = 3\pi/4,\ 7\pi/4}$$

$$\tan^2 x + 4 = 2\underbrace{\sec^2 x} + \tan x$$

$$\tan^2 x + 4 = 2(\tan^2 x + 1) + \tan x$$

$$\tan^2 x + 4 = 2\tan^2 x + 2 + \tan x$$

$$0 = \tan^2 x + \tan x - 2$$

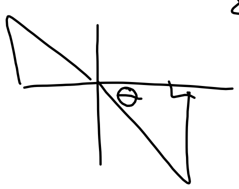
$$0 = (\tan x + 2)(\tan x - 1)$$

$$\tan x = -2 \qquad \tan x = 1$$

$$x = \qquad\qquad x = \pi/4,\ 5\pi/4$$

$$\tan^{-1}(-2) = \theta + 2\pi \in QIV$$

$$\& \ \theta + \pi \in QII$$

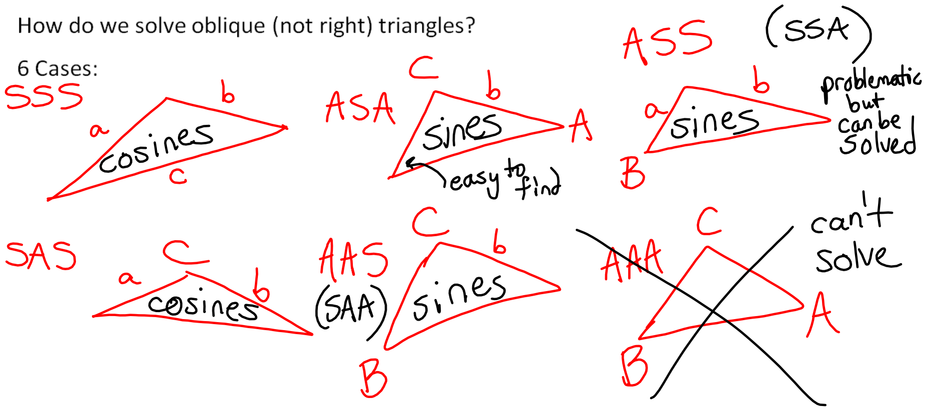


$$-\frac{\pi}{2} < \theta < 0$$

~~7~~8.1 The Law of Sines

How do we solve oblique (not right) triangles?

6 Cases:



The Law of Sines

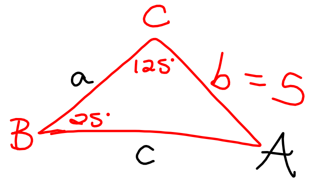
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

or

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

8.1

2. $B = 25°$, $C = 125°$, $b = 5$



AAS ⇒ Law of Sines

$A = 180° - 125° - 25°$

$\boxed{A = 30°}$

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{a}{\sin 30°} = \frac{5}{\sin 25°}$$

$$a = \frac{5 \sin 30°}{\sin 25°}$$

$a = (5 * \sin(30))/\sin(25)$

$\boxed{a \approx 5.92}$

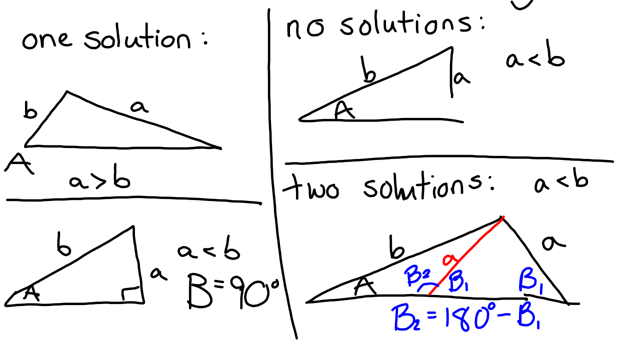
$$\frac{c}{\sin C} = \frac{b}{\sin B}$$

$$\frac{c}{\sin 125°} = \frac{5}{\sin 25°}$$

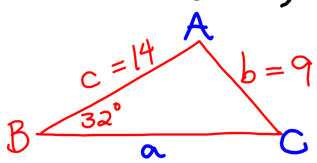
$$c = \frac{5 \sin 125°}{\sin 25°} \approx \boxed{9.69 \approx c}$$

8. $B = 54.8°$, $C = 72.6°$, $a = 14.4$

ASS, The Problematic Triangle



14. $B = 32°$, $c = 14$, $b = 9$



ASS, $b < c \Rightarrow$ problematic
0, 1, or 2 solutions

$$\frac{\sin C}{14} = \frac{\sin 32°}{9}$$

$$\sin C = \frac{14 \sin 32°}{9}$$

$$C = \sin^{-1}\left(\frac{14 \sin 32°}{9}\right)$$