The Law of Sines

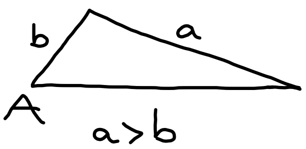
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

or

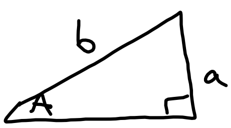
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

ASS, The Problematic Triangle

one solution:



$a > b$

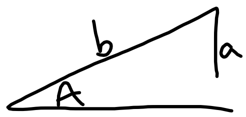


$a < b$

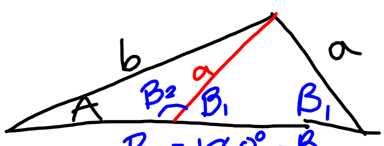
$B = 90°$

no solutions:

$a < b$

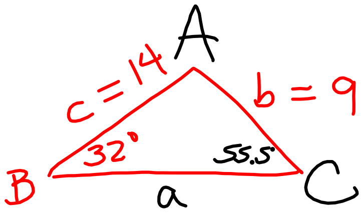


two solutions: $a < b$



$B_2 = 180° - B_1$

14. $B = 32^\circ$, $c = 14$, $b = 9$



ASS, $b < c \Rightarrow 0, 1,$ or 2 solutions

$$\frac{\sin C}{c} = \frac{\sin B}{b}$$

$$\sin C = \frac{c \sin B}{b}$$

$$C = \sin^{-1}\left(\frac{14 \sin 32^\circ}{9}\right) \approx \boxed{55.5^\circ = C}$$

$$A = 180^\circ - 32^\circ - 55.5^\circ$$

$$\boxed{A = 92.5^\circ}$$

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$a = \frac{b \sin A}{\sin B} = \frac{9 \sin 92.5^\circ}{\sin 32^\circ}$$

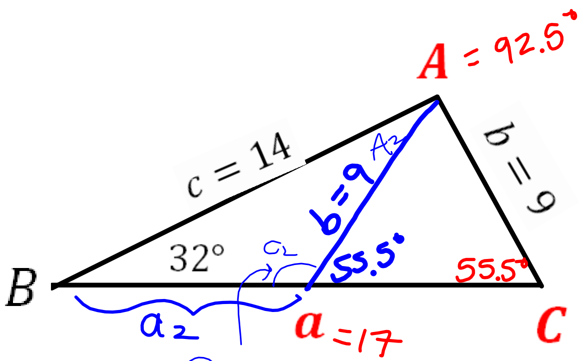
$$\boxed{a \approx 17}$$

<u>7.1 The Law of Sines</u>, continued

ASS – Problematic Triangle

14. $B = 32^\circ$, $c = 14$, $b = 9$

<u>Case 1</u>: $C \approx 55.5^\circ$, $A \approx 92.5^\circ$, $a \approx 17$



$$C_2 = 180^\circ - 55.5^\circ$$

$$\boxed{C_2 = 124.5^\circ}$$

$$A_2 = 180^\circ - 32^\circ - 124.5^\circ$$

$$\boxed{A_2 = 23.5^\circ}$$

$$a_2 = \frac{9 \sin 23.5^\circ}{\sin 32^\circ}$$

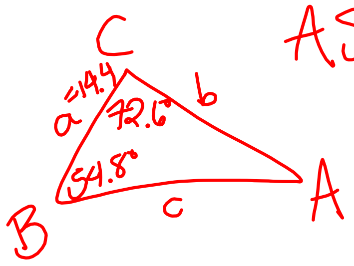
$$\frac{a_2}{\sin 23.5^\circ} = \frac{9}{\sin 32^\circ}$$

$$\boxed{a_2 = 6.8}$$

8. $B = 54.8^\circ$, $C = 72.6^\circ$, $a = 14.4$

Find side b.

ASA



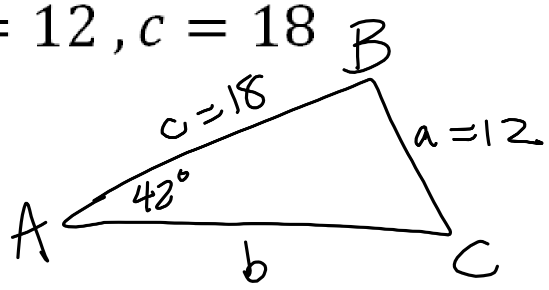
$A = 180^\circ - 54.8^\circ - 72.6^\circ$

$\boxed{A = 52.6^\circ}$

$$\frac{b}{\sin 54.8^\circ} = \frac{14.4}{\sin 52.6^\circ} \Rightarrow b = \frac{14.4 \sin 54.8^\circ}{\sin 52.6^\circ} = \boxed{14.8 = b}$$

16. $A = 42^\circ$, $a = 12$, $c = 18$

Find angle B.



ASS

$a < c$

0, 1, or 2 solutions

$$\frac{\sin C}{18} = \frac{\sin 42^\circ}{12}$$

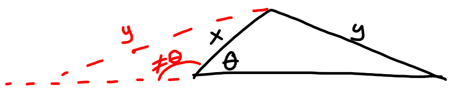
$$\sin C = \frac{18 \sin 42^\circ}{12} > 1$$

$$C = \sin^{-1}\left(\frac{18 \sin 42^\circ}{12}\right) = \text{undefined}$$

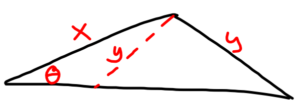
This is not a

18. $B = 22.6°, b = 5.55, a = 13.8$

Why does this ASS triangle have only one solution?



The measure of θ and the lengths of x & y are fixed. If we try to reposition y, the measure of θ changes, unlike in the 2-solution case:

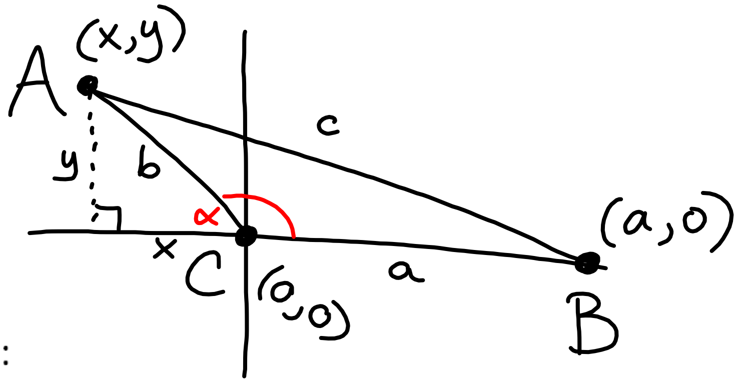


7.2 - The Law of Cosines

Derivation:

$\cos C = \frac{x}{b}$

$\sin C = \frac{y}{b}$



Distance Formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The Law of Cosines

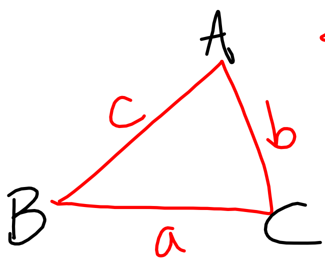
$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

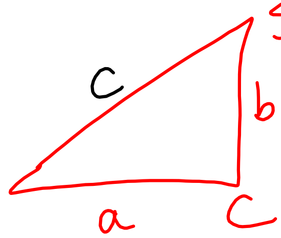
$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$c = \sqrt{a^2 + b^2 - 2ab\cos C}$$

SSS



SAS



7.2

16. $a = 60, b = 88, c = 120.$ $\underline{B = ?}$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$2ac\cos B = a^2 + c^2 - b^2$$

$$\cos^{-1}(\cos B) = B$$

$$\cos B = \frac{a^2+c^2-b^2}{2ac}$$

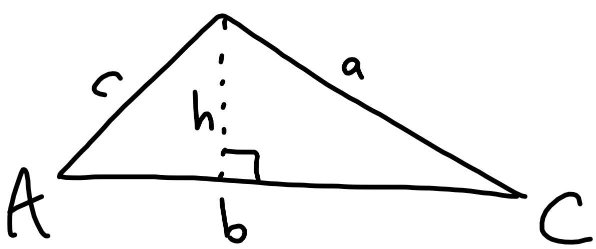
$$0^\circ < B < 180^\circ$$

$$B = \cos^{-1}\left(\frac{a^2+c^2-b^2}{2ac}\right)$$

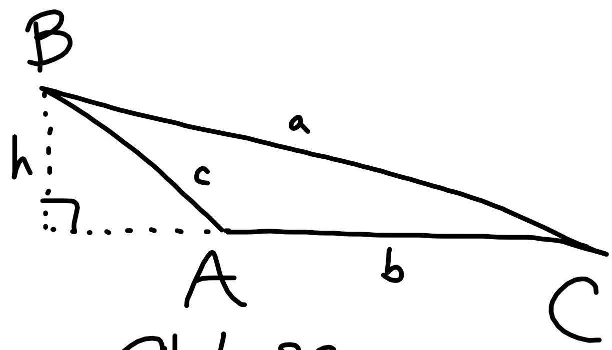
$$= \cos^{-1}\left(\frac{60^2+120^2-88^2}{2(60)(120)}\right) \approx \boxed{44.6^\circ = B}$$

$\cos^{-1}((60\text{^}2+120\text{^}2-88\text{^}2)/(2*60*120))$

7.1/7.2 Area of a Triangle



Acute



Obtuse

$$\sin A = \frac{h}{c} \qquad \sin C = \frac{h}{a}$$

$$c\sin A = h = a\sin C$$

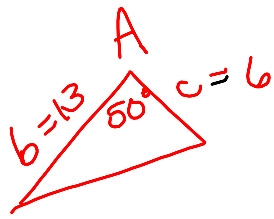
$$\text{area} = \frac{1}{2}\text{base} * \text{height}$$

$$= \frac{1}{2}bc\sin A$$

$$= \frac{1}{2}ba\sin C$$

Find the area of the triangle.

$A = 50°, b = 13\ cm, c = 6\ cm$



$\text{area} = \frac{1}{2}(13)(6)\sin 50°$

$\approx \boxed{29.9\ cm^2}$

How many solutions does each of these triangles have?

1. a=2.53 , b=3.76 , c=8.04 — SSS — 0

   fails triangle inequality
   $a+b<c$

2. A=15°, a=4 , c=11 — ASS — 2

   confirm by solving for $\angle C$ to see that it exists & is not $=90°$

3. A=72°, b=8.4 , c=17.2 — SAS — 1

   always only 1 sol'n for SAS

4. B=64°, b=2 , c=17 — ASS — 0

   confirm by failing to find $\angle C$

5. C=23°, b=4.9 , c=9.8 — ASS — 1

   if given opposite side is longer than adjacent side, always only 1 solution!

Prove the identity: 6.3 #87

Solve the equation: 6.6 #74