1.6 The Algebra of Functions

Given f & g, what are $(f+g)(x)$, $(f-g)(x)$, $(fg)(x)$, $\left(\frac{f}{g}\right)(x)$, $(f\circ g)(x)$, $(g\circ f)(x)$

$f(x) = x^2 - x$; $g(x) = x+1$

$(f+g)(x) = f(x)+g(x) = x^2 - x + x + 1 = \boxed{x^2+1}$

domain: $\mathbb{R}$; range: $[1,\infty)$

$(f-g)(x) = f(x)-g(x) = x^2 - x - (x+1) = \boxed{x^2-2x-1}$

domain: $\mathbb{R}$; range: too complicated for now

$(fg)(x) = f(x)\cdot g(x) = (x^2-x)(x+1) = x^3+x^2-x^2-x = \boxed{x^3-x}$

domain: $\mathbb{R}$; range: $\mathbb{R}$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x^2-x}{x+1}$

domain: $\{x \mid x\neq -1\}$; range: idk omg!

$(f\circ g)(x) = f(g(x)) = (x+1)^2 - (x+1) = x^2+2x+1-x-1 = x^2+x$

domain: $\mathbb{R}$; range: not obvious

$(g\circ f)(x) = g(f(x)) = (x^2-x)+1 = x^2-x+1$

domain: $\mathbb{R}$; range: ?

32. $f(x) = \sqrt{x+6}$; $g(x) = \frac{1}{x}$

$(f+g)(x) = \sqrt{x+6} + \frac{1}{x}$

$x+6\geq 0$, $x\geq -6$; $x\neq 0$

domain: $[-6,0)\cup(0,\infty)$

$(fg)(x) = \frac{\sqrt{x+6}}{x}$ domain: same

$\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x+6}}{\frac{1}{x}} = x\sqrt{x+6}$ domain: same

$f+g$, $f-g$, fg, $\frac{f}{g}$* have domains = intersection of domain of f & domain of g

* f/g also excludes values of x that make $g(x)=0$

$f(x) = x+2$; $g(x) = x$; $\left(\frac{f}{g}\right)(x) = \frac{x+2}{x}$ domain: $\{x \mid x\neq 0\}$

$f(x)=\sqrt{x+6}$; $g(x)=\frac{1}{x}$

$(f\circ g)(x)=\sqrt{\frac{1}{x}+6\cdot\frac{x}{x}}=\sqrt{\frac{1+6x}{x}}$ domain: $\frac{1+6x}{x}\geq 0$

$(g\circ f)(x)=\frac{1}{\sqrt{x+6}}$ domain: $(-6,\infty)$

$x+6>0$

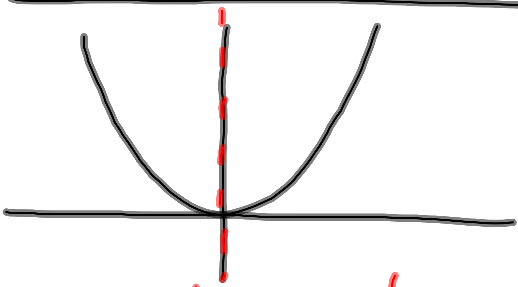
$x>-6$

84. $h(x)=|9x^2-4|$

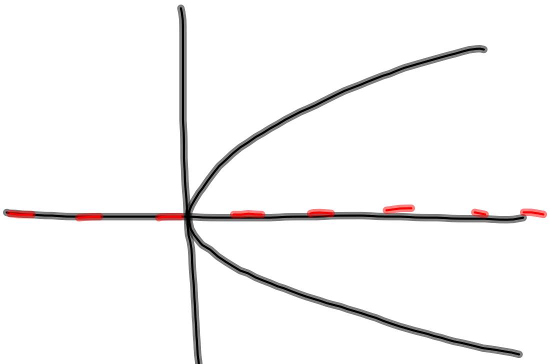
find functions f & g such that $h(x)=(f\circ g)(x)$.

$f(x)$	$g(x)$	
$\lvert 9x^2-4\rvert$	x	trivial cases
x	$\lvert 9x^2-4\rvert$	
$\lvert 9x-4\rvert$	x^2	
$\lvert x-4\rvert$	$9x^2$	
~~$3x-2$~~	~~$3x+2$~~	~~$3(3x+2)-2=9x+4$~~
$\lvert 3x-4\rvert$	$3x^2$	$\lvert 3(3x^2)-4\rvert=\lvert 9x^2-4\rvert$
$\lvert x^2-4\rvert$	$3x$	$\lvert (3x)^2-4\rvert=\lvert 9x^2-4\rvert$
$\lvert x\rvert$	$9x^2-4$	

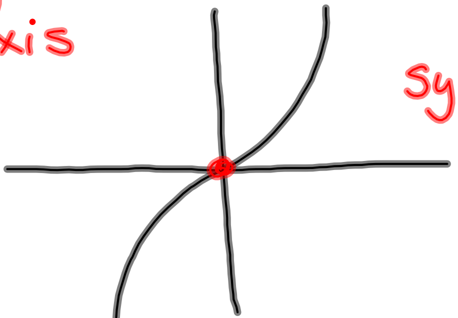
1.7 Symmetry & Transformations



symmetry w.r.t. the y-axis



Symmetry w.r.t. x-axis



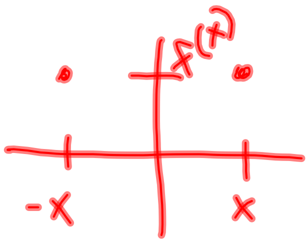
Symmetry w.r.t. origin

Even / Odd Functions

A function is <u>even</u> if

$$f(-x) = f(x)$$

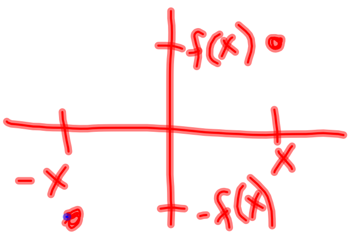
even functions are symmetric w.r.t. the y-axis

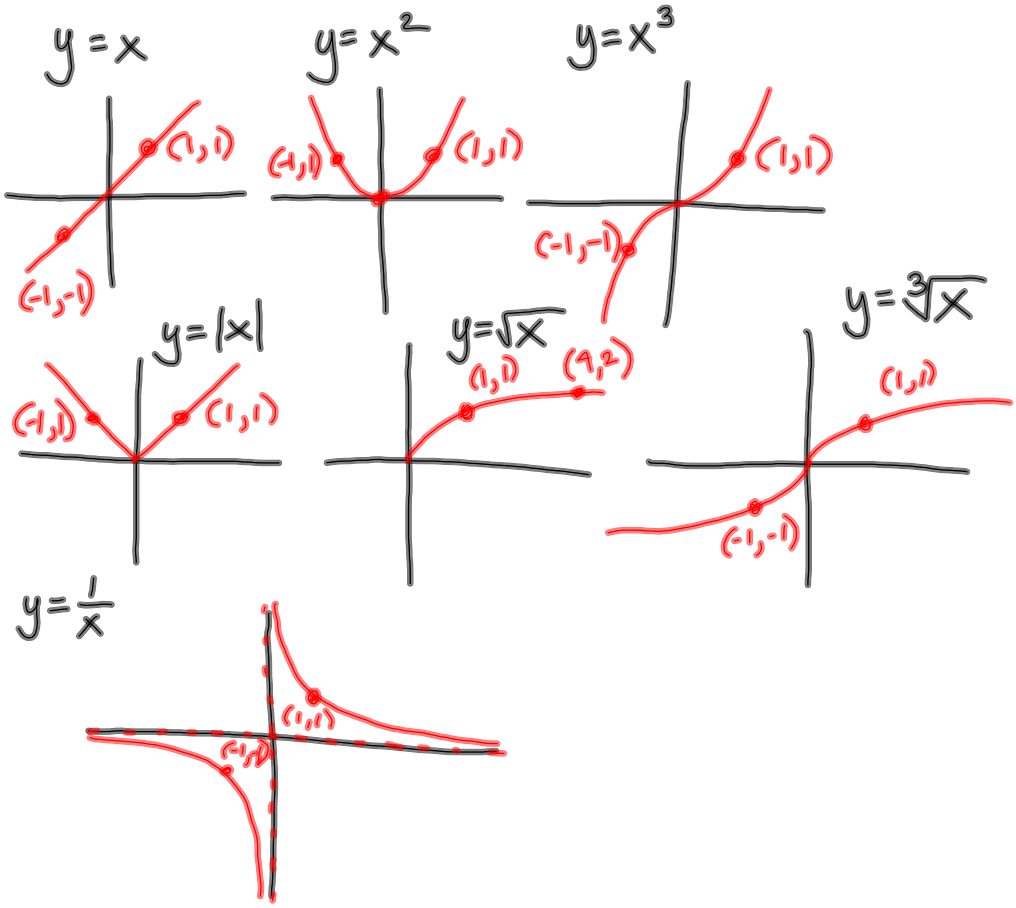


A function is <u>odd</u> if

$$f(-x) = -f(x)$$

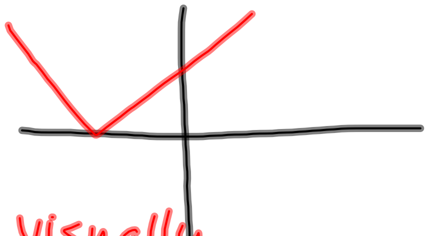
odd functions are symmetric w.r.t. the origin





1.7

8. $y=|x+5|$



visually, no symmetry

x-axis: $-y=|x+5|$

$y=-|x+5|$ no.

y-axis: $y=|-x+5|$ no.

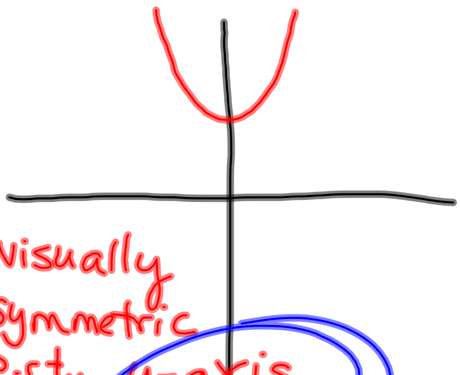
origin: $-y=|-x+5|$

$y=-|-x+5|$ no.

12. $x^2+4=3y$

$3y=x^2+4$

$y=\frac{1}{3}x^2+\frac{4}{3}$



visually symmetric w.r.t. y-axis

x-axis: $x^2+4=3(-y)$

$-x^2-4=3y$ no.

y-axis: $(-x)^2+4=3y$

$x^2+4=3y$ yes

origin: $(-x)^2+4=3(-y)$

$-x^2-4=3y$ no.

42. $f(x)=x+\frac{1}{x}$

even/~~odd~~/neither?

$f(-x)=-x+\frac{1}{-x}$

$=-\left(x+\frac{1}{x}\right)$

$=-f(x)$

$\Rightarrow f$ is $\boxed{\text{odd}}$

& symmetric w.r.t. origin

40. $f(x) = 7x^3 + 4x - 2$

even/odd/neither?

$$f(-x) = 7(-x)^3 + 4(-x) - 2$$
$$= -7x^3 - 4x - 2$$

$\Rightarrow$ f is $\boxed{\text{neither}}$ even nor odd

1.6 # 23, 29, 31; 71, 75; 81

1.7 # 9, 11, 39-47 odd