1.6 The Algebra of Functions

Given f & g, what are $(f+g)(x)$, $(f-g)(x)$, $(fg)(x)$, $\left(\frac{f}{g}\right)(x)$, $(f\circ g)(x)$, $(g\circ f)(x)$

$f(x) = x^2 - x$; $g(x) = x + 1$

$(f+g)(x) = f(x) + g(x) = x^2 - x + x + 1 = \boxed{x^2+1}$

domain: $\mathbb{R}$; range: $[1, \infty)$

$(f-g)(x) = f(x) - g(x) = x^2 - x - (x+1) = \boxed{x^2 - 2x - 1}$

domain: $\mathbb{R}$; range: not as easy

$(fg)(x) = f(x)\cdot g(x) = (x^2 - x)(x+1) = x^3 + x^2 - x^2 - x = \boxed{x^3 - x}$

domain: $\mathbb{R}$; range: $\mathbb{R}$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \boxed{\frac{x^2-x}{x+1}}$

domain: $\{x \mid x \neq -1\}$; range: not obvious

$(f\circ g)(x) = f(g(x)) = (x+1)^2 - (x+1) = x^2 + 2x + 1 - x - 1$
$= \boxed{x^2 + x}$

domain: $\mathbb{R}$

$(g\circ f)(x) = g(f(x)) = (x^2 - x) + 1 = \boxed{x^2 - x + 1}$

domain: $\mathbb{R}$

32. $f(x) = \sqrt{x+6}$; $g(x) = \frac{1}{x}$

$(f+g)(x) = \sqrt{x+6} + \frac{1}{x}$ domain:

$x + 6 \geq 0$
$x \geq -6$ $x \neq 0$ $[-6, 0) \cup (0, \infty)$

$(fg)(x) = \frac{\sqrt{x+6}}{x}$ domain: $[-6, 0) \cup (0, \infty)$

$\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x+6}}{\frac{1}{x}} = x\sqrt{x+6}$ domain: same

$f+g$, $f-g$, fg, $\frac{f}{g}$* have domains
= intersection of domain of f & domain of g

* f/g also excludes values of x that make
$g(x) = 0$

$f(x) = x+2$; $g(x) = x$; $\left(\frac{f}{g}\right)(x) = \frac{x+2}{x}$ domain: $\{x \mid x \neq 0\}$

$f(x)=\sqrt{x+6}$; $g(x)=\frac{1}{x}$

$(f\circ g)(x)=\sqrt{\frac{1}{x}+6\cdot\frac{x}{x}}=\sqrt{\frac{1+6x}{x}}$ domain: $\frac{1+6x}{x}\geq 0$

$(g\circ f)(x)=\frac{1}{\sqrt{x+6}}$ domain: $\{x \mid x>-6\}$

$x+6>0$

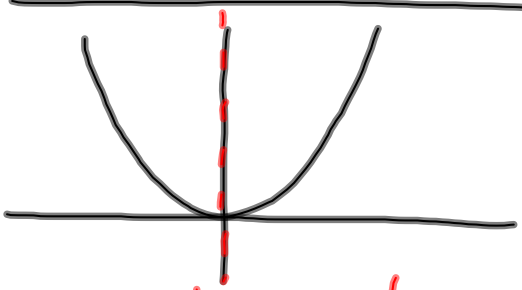
$x>-6$

84. $h(x)=|9x^2-4|$

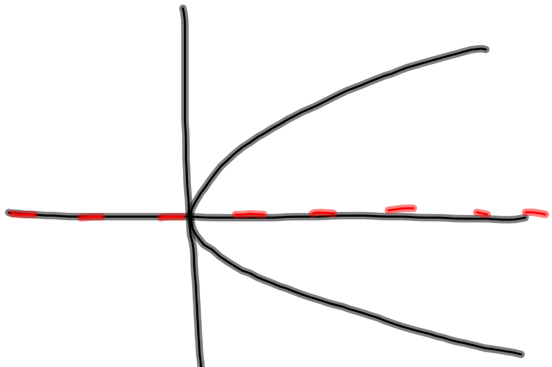
find functions f & g such that $h(x)=(f\circ g)(x)$.

	$f(x)$	$g(x)$	
	$\lvert 9x^2-4\rvert$	x	trivial cases
	x	$\lvert 9x^2-4\rvert$	
	$\lvert x\rvert$	$9x^2-4$	
$h(x)$: $\lvert 9x^2-4\rvert$	$\cancel{9x^2-4}$	$\cancel{\lvert x\rvert}$	$\cancel{9\lvert x\rvert^2-4}$
	$\cancel{x^2}$	$\cancel{3x-2}$	$\cancel{(3x-2)^2=9x^2-12x+4}$
	$\lvert 9x-4\rvert$	x^2	
	$\lvert 2x\rvert$	$\frac{9}{2}x^2-2$	
	$\lvert x-4\rvert$	$9x^2$	
	$\lvert 9x\rvert$	$x^2-\frac{4}{9}$	$\cancel{\lvert 9(x^2-4)\rvert=\lvert 9x^2-36\rvert}$

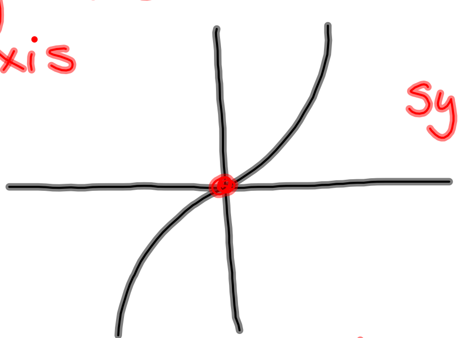
1.7 Symmetry & Transformations



symmetry w.r.t. the y-axis



Symmetry w.r.t. x-axis

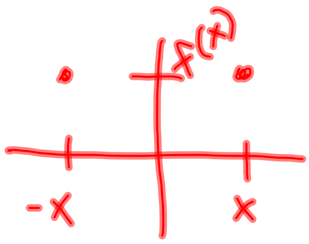


Symmetry w.r.t. origin

Even / Odd Functions

A function is <u>even</u> if

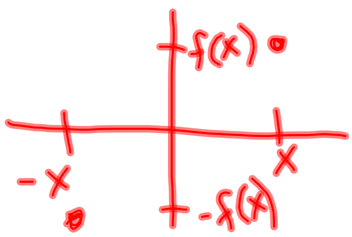
$$f(-x) = f(x)$$



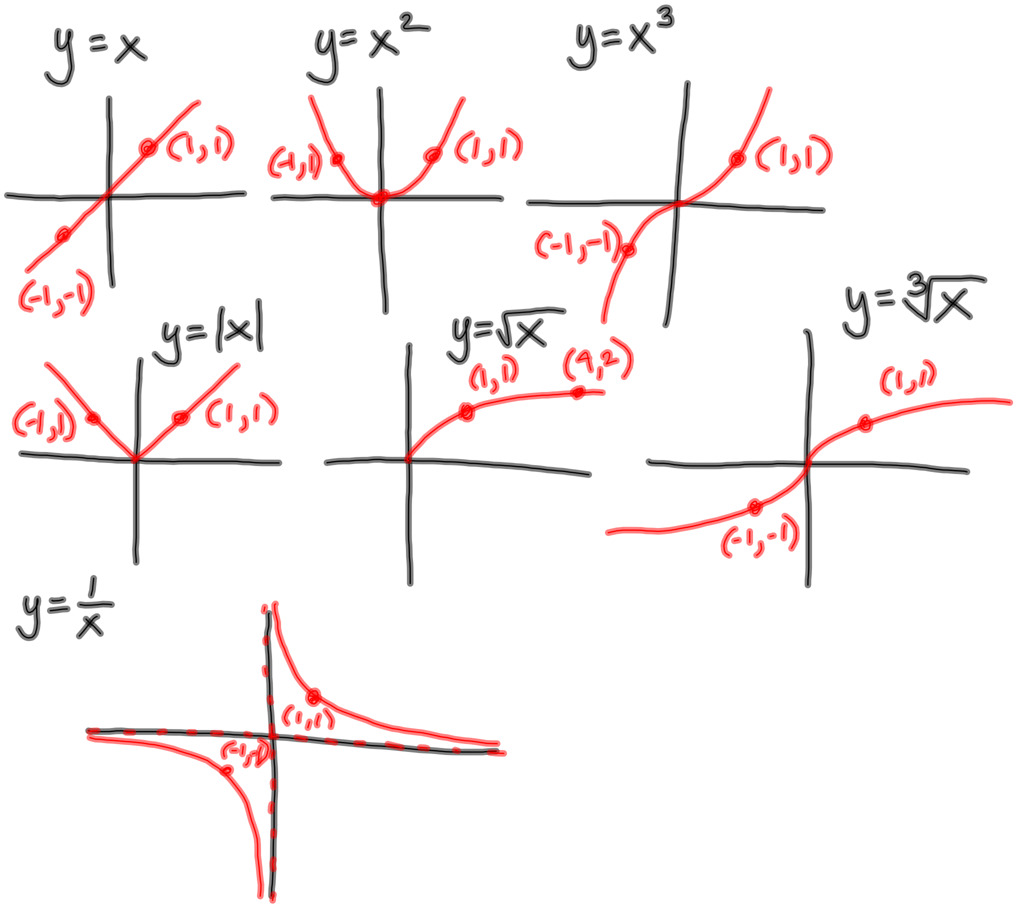
even functions are symmetric w.r.t. the y-axis

A function is <u>odd</u> if

$$f(-x) = -f(x)$$

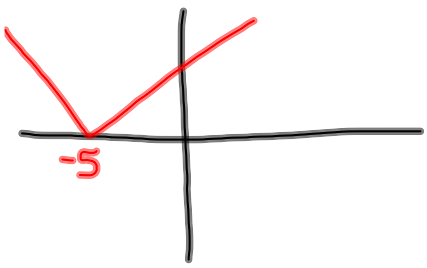


odd functions are symmetric w.r.t. the origin



1.7

8. $y=|x+5|$



x-axis: $-y=|x+5|$

$y=-|x+5|$ no!

y-axis: $y=|-x+5|$ no!

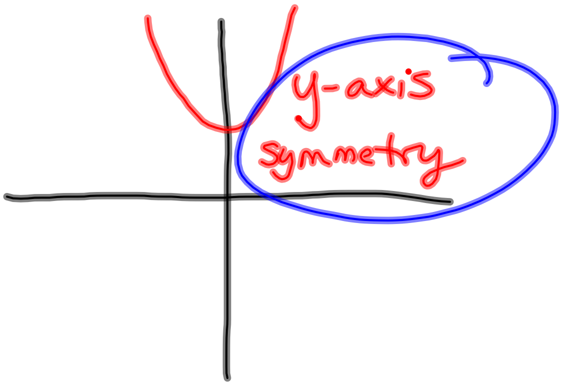
origin: $-y=|-x+5|$

$y=-|-x+5|$ no!

not symmetric w.r.t. x or y axis or origin

12. $x^2 + 4 = 3y$

$y = \frac{1}{3}x^2 + \frac{4}{3}$



x-axis: $x^2+4 = 3(-y)$

$-x^2-4 = 3y$ no.

y-axis: $(-x)^2+4 = 3y$

$x^2+4 = 3y$ yes.

origin: $(-x)^2+4 = 3(-y)$

$-x^2-4 = 3y$ no.

42. $f(x) = x + \frac{1}{x}$

even/odd/neither?

$f(-x) = -x + \frac{1}{-x}$

$= -\left(x + \frac{1}{x}\right)$

$= -f(x)$

$\Rightarrow$ f is odd.

(& symmetric w.r.t. origin)

40. $f(x) = 7x^3 + 4x - 2$

even/odd/neither?

$f(x) = 7(-x)^3 + 4(-x) - 2$

$= -7x^3 - 4x - 2$

f is neither even nor odd

1.6 #23, 29, 31; 71, 75; 81

1.7 #9, 11, 39-47 odd