Quiz 2 Solutions

1. State the equation of:

a. the *horizontal* line that passes through the point $(-5, 2)$. (1pt)

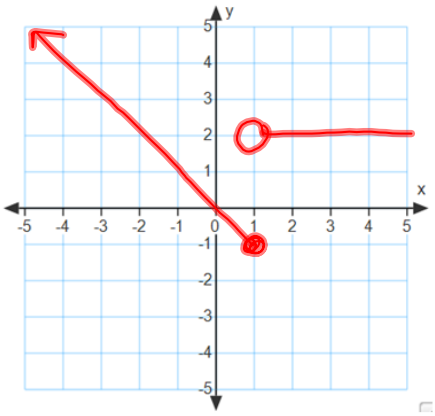
$y = 2$

b. the *vertical* line that passes through the point $(5, -2)$. (1pt)

$x = 5$

2. *Graph* the piecewise function. (3 pts)

$$f(x) = \begin{cases} -x, & x \le 1 \\ 2, & x > 1 \end{cases}$$



3. State the *domain* and *range* of the piecewise function given in #2. (1pt ea.)

a. domain:

$\mathbb{R} = (-\infty, \infty)$

b. range:

$[-1, \infty)$

For the functions $f(x) = \frac{1}{x}$ and $g(x) = 3 - x$,

4. (1pt ea.)

a. find $(f \circ g)(x)$ $= \frac{1}{3-x}$

b. give the domain of $(f \circ g)(x)$

$\{x \mid x \neq 3\} = (-\infty, 3) \cup (3, \infty)$

5. (1pt ea.)

a. find $(g + f)(x)$

$\frac{1}{x} + 3 - x = \frac{-x^2 + 3x + 1}{x}$

b. give the domain of $(g + f)(x)$

$\{x \mid x \neq 0\} = (-\infty, 0) \cup (0, \infty)$

6. The graph of an equation is symmetric with respect to: (1pt ea., must have both blanks correct to get point)

a. *the y-axis* if replacing x with $-x$ yields the original equation.

b. *the origin* if replacing x & y with $-x$ & $-y$ yields the original equation.

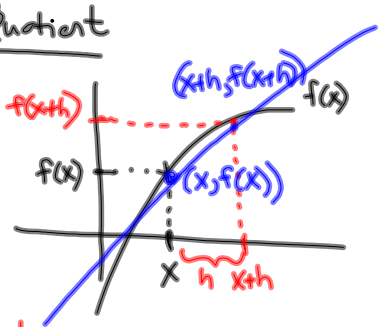
7. A function *f* is (1pt ea.)

a. *even* if $f(-x) =$ $f(x)$.

b. *odd* if $f(-x) =$ $-f(x)$.

1.6 The Difference Quotient

$$\frac{f(x+h)-f(x)}{h}$$



$f(x)=3x^2+2x-1$

$f(x+h)=3(x+h)^2+2(x+h)-1$

$m=\frac{f(x+h)-f(x)}{x+h-x}$

$$\frac{f(x+h)-f(x)}{h}=\frac{3(x+h)^2+2(x+h)-1-(3x^2+2x-1)}{h}$$

$$=\frac{3(x^2+2xh+h^2)+2x+2h-1-3x^2-2x+1}{h}$$

$$=\frac{3x^2+6xh+3h^2+2h-3x^2}{h}$$

$$=\frac{h(6x+3h+2)}{h}=\boxed{6x+3h+2}$$

2.3 Solving equations reducible to a quadratic

$ax^2+bx+c=0$

94. $y^6-26y^3-27=0$

Let $y^3=x$, $(y^3)^2=y^6=x^2$

$x^2-26x-27=0$

$(x-27)(x+1)=0$

$x=27$ $\quad x=-1$

$y^3=27$ $\quad y^3=-1$

$y=\sqrt[3]{27}$ $\quad y=\sqrt[3]{-1}$

$\boxed{y=3 \quad y=-1}$

100. $x^{1/2} - 4x^{1/4} = -3$

$x^{1/2} - 4x^{1/4} + 3 = 0$

Let $u = x^{1/4}$; $u^2 = (x^{1/4})^2 = x^{1/2}$

$u^2 - 4u + 3 = 0$

$(u-1)(u-3) = 0$

$u = 1 \qquad u = 3$

$(x^{1/4})^4 = (1)^4 \qquad (x^{1/4})^4 = (3)^4$

$x = 1 \qquad x = 81$

2.4 Analyzing Graphs of Quadratic Functions

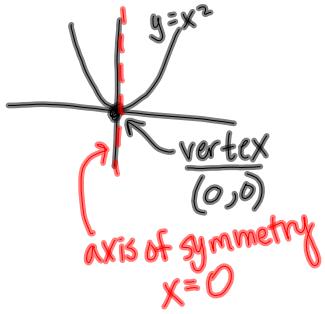
$f(x) = ax^2 + bx + c$ ← standard form

The graph of a quadratic function is a parabola.

$f(x) = a(x-h)^2 + k$

Vertex: (h, k)

axis of symmetry: (vertical line through the vertex) $x = h$



If $a > 0$ opens up, vertex is a minimum

incr: (h, ∞)

decr: $(-\infty, h)$

If $a < 0$ opens down, vertex is a maximum

incr: $(-\infty, h)$

decr: (h, ∞)

2.4

8. $f(x) = x^2 + 2x + 6$

$= (x^2 + 2x + 1) + 6 - 1$

$f(x) = (x+1)^2 + 5$

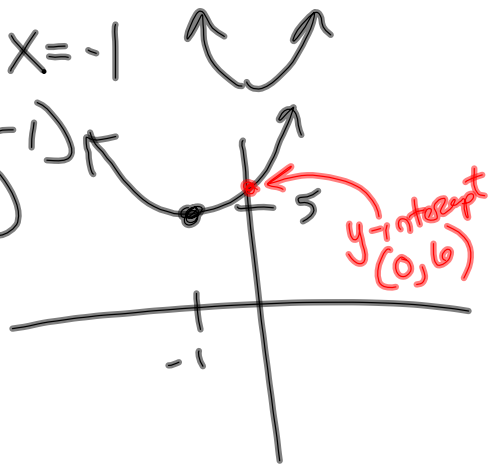
$f(x) = a(x-h)^2 + k$
vertex: (h,k)

vertex: $(-1,5)$ min.

axis of symmetry: $x = -1$

decreasing on: $(-\infty, -1)$

increasing on: $(-1, \infty)$



12. $f(x) = 2x^2 - 10x + 14$

$\frac{1}{2}(-5) = \frac{-5}{2}$

$(-\frac{5}{2})^2 = \frac{25}{4}$

$= 2(x^2 - 5x + (\frac{-5}{2})^2) + 14 - 2(\frac{25}{4})$

$\frac{28}{2} - \frac{25}{2}$

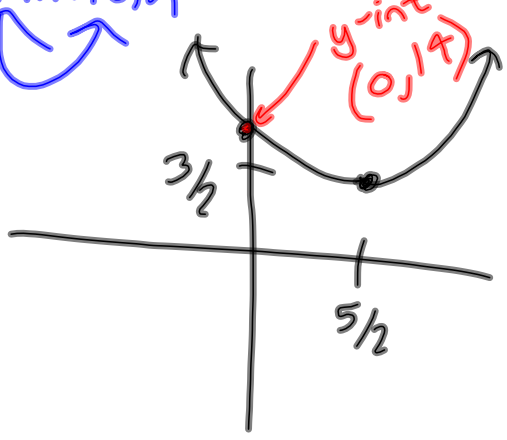
$f(x) = 2(x - \frac{5}{2})^2 + \frac{3}{2}$

vertex: $(\frac{5}{2}, \frac{3}{2})$ minimum

axis of sym: $x = 5/2$

decreasing: $(-\infty, 5/2)$

increasing: $(5/2, \infty)$



14. $f(x) = -3x^2 - 3x + 1$

to get x-intercepts, solve $-3x^2 - 3x + 1 = 0$

$x = \frac{3 \pm \sqrt{9 - 4(+3)(1)}}{2(-3)}$

$= \frac{3 \pm \sqrt{21}}{-6} = -\frac{1}{2} + \frac{\sqrt{21}}{6}; -\frac{1}{2} - \frac{\sqrt{21}}{6}$

$= -3\left(x^2 + x + \left(\frac{1}{2}\right)^2\right) + 1 - \left[-3\left(\frac{1}{2}\right)^2\right]$

$\frac{4}{4} + \frac{3}{4}$

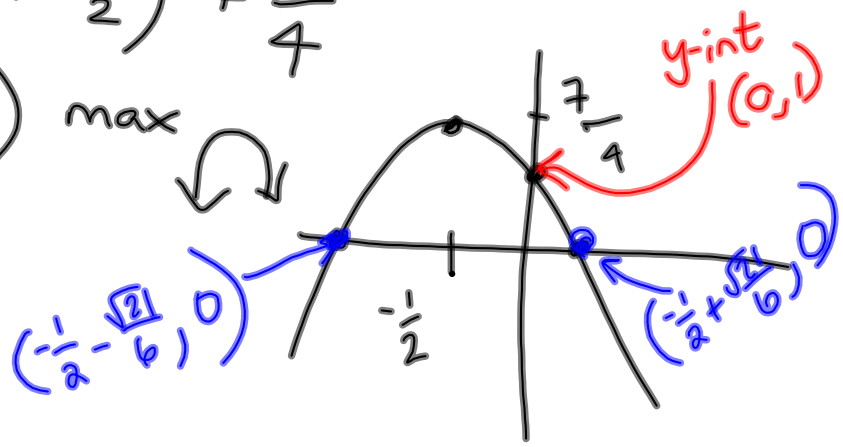
$f(x) = -3\left(x + \frac{1}{2}\right)^2 + \frac{7}{4}$

vertex: $\left(-\frac{1}{2}, \frac{7}{4}\right)$ max

axis: $x = -\frac{1}{2}$

incr: $\left(-\infty, -\frac{1}{2}\right)$

decr: $\left(-\frac{1}{2}, \infty\right)$



HW

1.6 # 41,43,45

2.3 # 89,93,97,99,103

2.4 # 3-13 odd