Quiz 2 Solutions

1. State the equation of:

a. the *horizontal* line that passes through the point $(2, -5)$. (1pt)

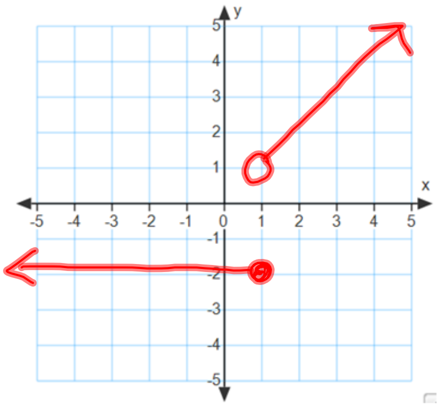
$y = -5$

b. the *vertical* line that passes through the point $(2, -5)$. (1pt)

$x = 2$

2. *Graph* the piecewise function. (3 pts)

$$f(x) = \begin{cases} -2, & x \le 1 \\ x, & x > 1 \end{cases}$$



3. State the *domain* and *range* of the piecewise function given in #2. (1pt ea.)

a. domain:

$\mathbb{R} = (-\infty, \infty)$

b. range:

$\{-2\} \cup (1, \infty)$

For the functions $f(x) = \frac{1}{x}$ and $g(x) = x - 3$,

4. (1pt ea.)

a. find $(f \circ g)(x) = \dfrac{1}{x-3}$

b. give the domain of $(f \circ g)(x)$

$\{x \mid x \neq 3\} = (-\infty, 3) \cup (3, \infty)$

5. (1pt ea.)

a. find $(g + f)(x)$

$\frac{1}{x} + x - 3 = \dfrac{x^2 - 3x + 1}{x}$

b. give the domain of $(g + f)(x)$

$\{x \mid x \neq 0\} = (-\infty, 0) \cup (0, \infty)$

6. The graph of an equation is symmetric with respect to: (1pt ea., must have both blanks correct to get point)

a. *the y-axis* if replacing x with $-x$ yields the original equation.

b. *the origin* if replacing x & y with $-x$ & $-y$ yields the original equation.

7. A function f is (1pt ea.)

a. *even* if $f(-x) = f(x)$.

b. *odd* if $f(-x) = -f(x)$.

2.3

31. $x^2+8x+25=0$

$x^2+8x+16 = -25+16$

$(x+4)^2 = -9$

$x+4 = \pm\sqrt{-9}$

$x+4 = \pm 3i$

$\boxed{x=-4\pm 3i}$

$-4+3i$ & $-4-3i$

33. $3x^2+5x-2=0$

$3\left(x^2+\frac{5}{3}x\right) = 2$

$\frac{1}{2}\left(\frac{5}{3}\right) = \frac{5}{6}$

$\left(\frac{5}{6}\right)^2 = \frac{25}{36}$

$x^2+\frac{5}{3}x+\left(\frac{5}{6}\right)^2 = \frac{2}{3}\cdot\frac{12}{12}+\frac{25}{36}$

$\left(x+\frac{5}{6}\right)^2 = \frac{49}{36}$

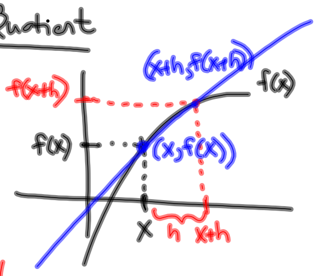
$x+\frac{5}{6} = \pm\frac{7}{6}$

$-\frac{5}{6}+\frac{7}{6}$ $\quad$ $-\frac{5}{6}-\frac{7}{6}$

$\frac{2}{6} = \frac{1}{3}$ $\quad$ $\frac{-12}{6} = -2$

1.6 The Difference Quotient

$$\frac{f(x+h)-f(x)}{h}$$



$$m=\frac{f(x+h)-f(x)}{\cancel{x}+h\cancel{-x}}$$

$f(x)=3x^2+2x-1$

$f(x+h)=3(x+h)^2+2(x+h)-1$

$$\frac{f(x+h)-f(x)}{h}=$$

$$\frac{3(x+h)^2+2(x+h)-1-(3x^2+2x-1)}{h}=$$

$$=\frac{3(x^2+2xh+h^2)+\cancel{2x}+2h\cancel{-1}-3x^2\cancel{-2x}\cancel{+1}}{h}=$$

$$=\frac{\cancel{3x^2}+6xh+3h^2+2h\cancel{-3x^2}}{h}$$

$$=\frac{\cancel{h}(6x+3h+2)}{\cancel{h}}$$

$$=\boxed{6x+3h+2}$$

2.3 Solving equations reducible to a quadratic

$ax^2+bx+c=0$

94. $y^6-26y^3-27=0$

Let $x=y^3$; $x^2=(y^3)^2=y^6$

$x^2-26x-27=0$

$(x-27)(x+1)=0$

$x=27$; $x=-1$

$y^3=27 \qquad y^3=-1$

$y=\sqrt[3]{27} \qquad y=\sqrt[3]{-1}$

$\boxed{y=3} \qquad \boxed{y=-1}$

100. $x^{1/2} - 4x^{1/4} = -3$

$x^{1/2} - 4x^{1/4} + 3 = 0$

$u = x^{1/4}$; $u^2 = (x^{1/4})^2 = x^{1/2}$

$(x^{\frac{1}{4}})^4 = (3)^4$

$u^2 - 4u + 3 = 0$

$(u-3)(u-1) = 0$

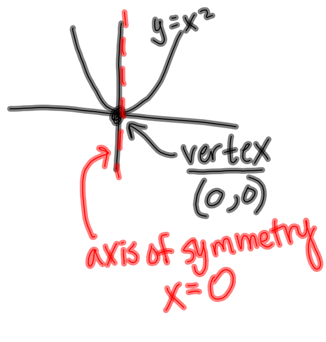
$x^{1/4} = u = 3, 1$

$x = 3^4, 1^4 = \boxed{81, 1}$

2.4 Analyzing Graphs of Quadratic Functions

$f(x) = ax^2 + bx + c$ ← standard form

The graph of a quadratic function is a parabola.



$f(x) = a(x-h)^2 + k$

vertex: (h, k)

axis of symmetry: (vertical line through the vertex) $x = h$

If $a > 0$ opens up, vertex is a minimum

Incr: (h, ∞)

decr: $(-\infty, h)$

If $a < 0$ opens down, vertex is a maximum

Incr: $(-\infty, h)$

decr: (h, ∞)

2.4

8. $f(x) = (x^2 + 2x) + 6$

$= \underbrace{x^2 + 2x + 1}\ \underbrace{+ 6 - 1}$

$f(x) = (x+1)^2 + 5$

vertex: $(-1, 5)$

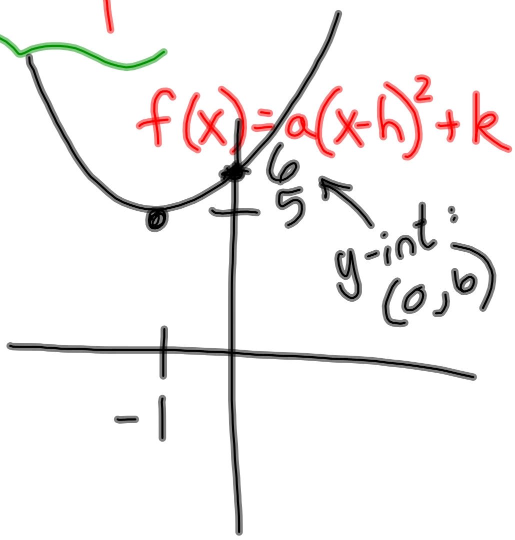
axis of symmetry: $x = -1$

minimum value @ $(-1, 5)$

incr: $(-1, \infty)$

decr: $(-\infty, -1)$

$f(x) = a(x-h)^2 + k$



12. $f(x) = (2x^2 - 10x) + 14$

$= 2\left(x^2 - 5x + \left(\frac{-5}{2}\right)^2\right) + 14 - 2\left(\frac{-5}{2}\right)^2$

$= 2\left(x - \frac{5}{2}\right)^2 + 14\cdot\frac{2}{2} - \cancel{2}\left(\frac{25}{\cancel{4}\,2}\right)$

$\frac{28}{2} - \frac{25}{2}$

$f(x) = 2\left(x - \frac{5}{2}\right)^2 + \frac{3}{2}$

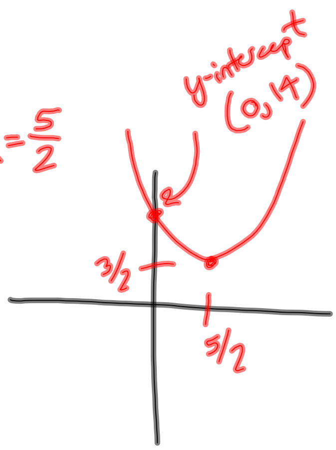
vertex: $\left(\frac{5}{2}, \frac{3}{2}\right)$

axis of symmetry: $x = \frac{5}{2}$

max/min:

incr: $(5/2, \infty)$

decr: $(-\infty, 5/2)$



14. $f(x) = -3x^2 - 3x + 1$

$= -3\left(x^2 + x + \left(\tfrac{1}{2}\right)^2\right) + 1 - \left[-3\left(\tfrac{1}{2}\right)^2\right]$

$= -3\left(x + \tfrac{1}{2}\right)^2 + 1\tfrac{4}{4} + \tfrac{3}{4}$

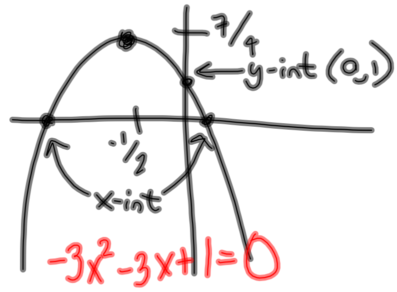
$f(x) = -3\left(x + \tfrac{1}{2}\right)^2 + \tfrac{7}{4}$

vertex: $\left(-\tfrac{1}{2}, \tfrac{7}{4}\right)$
max

axis of symm: $x = -\tfrac{1}{2}$

inc: $\left(-\infty, -\tfrac{1}{2}\right)$

dec: $\left(-\tfrac{1}{2}, \infty\right)$



$-3x^2 - 3x + 1 = 0$

$\dfrac{3 \pm \sqrt{9 + 4(+3)(1)}}{-6}$

x-int's $= \left(\dfrac{3 \pm \sqrt{21}}{-6}, 0\right)$ points $(x, 0)$

HW

1.6 # 41, 43, 45

2.3 # 89, 93, 97, 99, 103

2.4 # 3-13 odd