For the function $f(x) = 2x^2 - 2x - 4$,

1. Construct but do not simplify the difference quotient for the function. (2 points)

$$\frac{f(x+h)-f(x)}{h} = \boxed{\frac{2(x+h)^2-2(x+h)-4-(2x^2-2x-4)}{h}}$$

2. State the vertex. (2 point)

$$\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right) = \boxed{\left(\frac{1}{2}, \frac{-9}{2}\right)}$$

3. State the equation of the axis of symmetry. (2 points)

$$\boxed{x = \frac{1}{2}}$$

4. Solve the quadratic equation. $2x^2 - 2x - 4 = 0$. (2 points)

$$2(x^2-x-2)=0$$
$$2(x-2)(x+1)=0$$
$$\boxed{x = 2, -1}$$

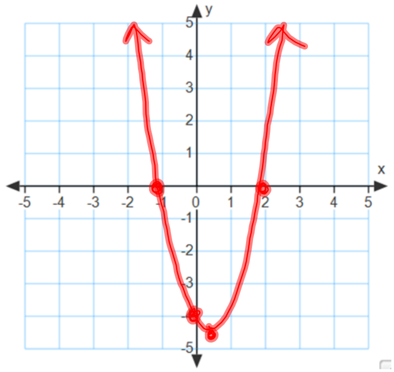
5. Determine the y-intercept for the function. (2 points)

$$f(0) = -4 \qquad \boxed{(0,-4)}$$

6. Determine the x-intercept(s) (if any). (2 points)

$$\boxed{(2,0)\ \&\ (-1,0)}$$

7. Graph the function. (3 points)



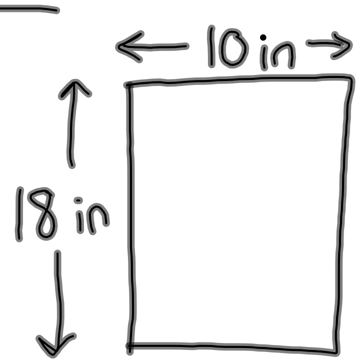
Bonus: State the equation of a function similar in shape to the greatest integer function, but which contains "steps" which are two units in length, the step $0 < x \le 2$ has y-value 0, and the step $2 < x \le 4$ has y-value 1.

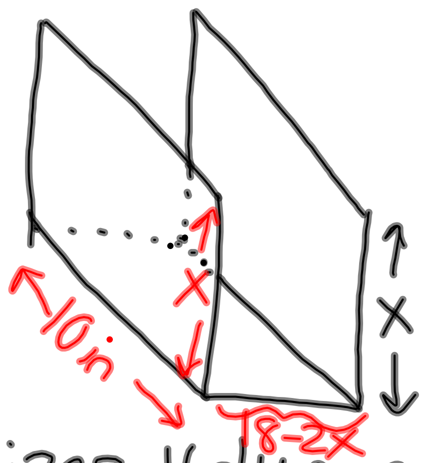
$$f(x) = \begin{cases} [\![\frac{1}{2}x]\!], & x \notin \mathbb{Z} \\ \frac{1}{2}x - 1, & x \in \mathbb{Z} \end{cases}$$

$$\mathbb{Z}$$

2.4

43.





what height x maximizes Volume $V(x)$

$$V(x) = 10x(18-2x)$$

$$V(x) = -20x^2 + 180x$$

Max height: $\frac{-b}{2a} = \frac{-18\cancel{0}}{2(-2\cancel{0})} = \boxed{\frac{9}{2}\text{ in}}$

49. $R(x) = 50x - 0.5x^2$
$C(x) = 10x + 3$

$P(x) = R(x) - C(x)$
$P(x) = 50x - 0.5x^2 - (10x + 3)$
$P(x) = -0.5x^2 + 40x - 3$

#units sold: $\frac{-b}{2a} = \frac{-40}{2(-0.5)} = \boxed{40 \text{ units}}$

Max Profit: $P(40) = -0.5(40)^2 + 40(40) - 3$
$\approx \boxed{\$797}$

15. $(4, -2)$ is on $y = f(x)$
what point is on $y = 3f(x)$?

$\boxed{(4, -6)}$

16. $x^2+4x=9$

$x^2+4x+\left(\frac{4}{2}\right)^2 = 9+4$

$(x+2)^2=13$

$x+2=\pm\sqrt{13}$

$\boxed{x=-2\pm\sqrt{13}}$

$f(x)=\underbrace{2x^2-6x}+5$

$=2\left(x^2-3x+\left(\frac{-3}{2}\right)^2\right)+5-2\left(\frac{3}{2}\right)^2$

$=2\left(x-\frac{3}{2}\right)^2+\frac{10}{2}-\frac{9}{2}$

$f(x)=2\left(x-\frac{3}{2}\right)^2+\frac{1}{2}$

vertex: $\left(\frac{3}{2},\frac{1}{2}\right)$

$\frac{-b}{2a}=\frac{3}{2}$; $f\left(-\frac{b}{2a}\right)=\frac{1}{2}$

even/odd/neither

$f(-x) = f(x)$ even

$f(-x) = -f(x)$ odd

$f(x) = -x^3 + \frac{1}{x^5}$

$f(-x) = -(-x)^3 + \frac{1}{(-x)^5} = x^3 - \frac{1}{x^5}$

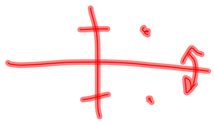
$= -\left(-x^3 + \frac{1}{x^5}\right)$

$= -f(x)$

$\Rightarrow f$ is odd

b. origin

symmetry tests



w.r.t.

x-axis   replace y w/ -y
    f(x) w/ -f(x)

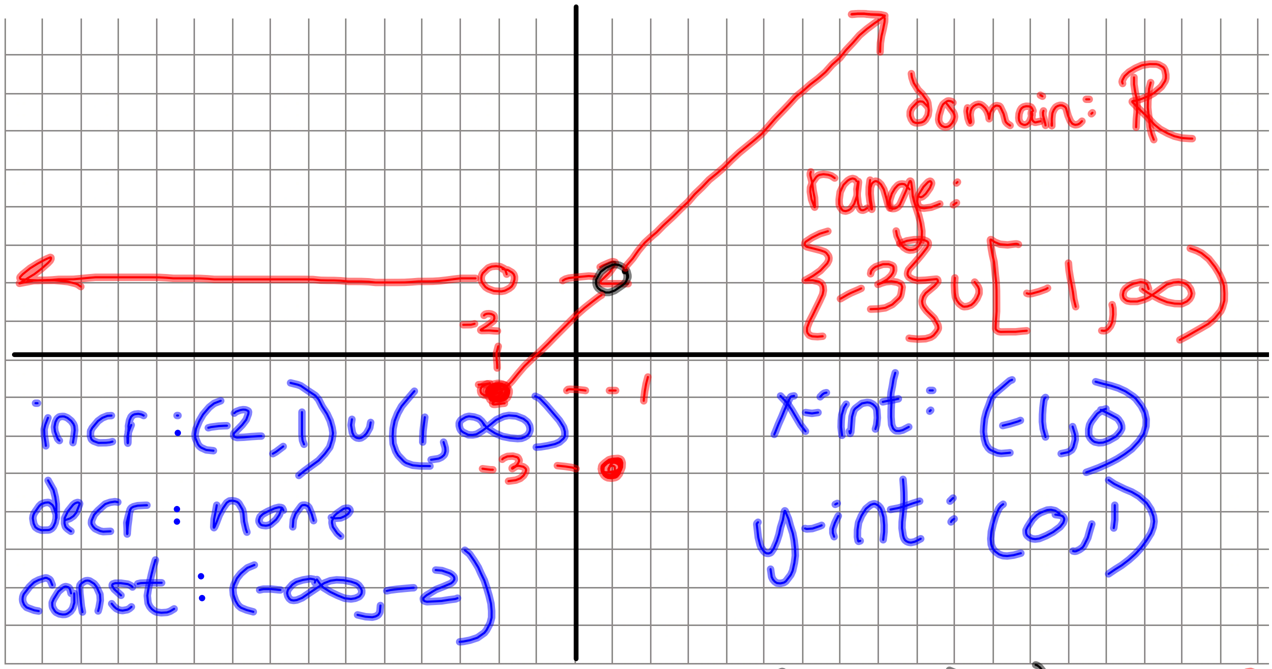
y-axis   x w/ -x

origin   x & y w/ -x & -y

If replacement yields original equation, then it has that symmetry.

e.g. $y = x^2 \Longleftrightarrow y = (-x)^2$

$y = x^3 \Longleftrightarrow -y = (-x)^3$



$$f(x) = \begin{cases} 2, & x < -2 \\ \cancel{\dfrac{x^2-1}{x-1}}, & x \geq -2, x \neq 1 \\ -3, & x = 1 \end{cases}$$

$$\frac{x^2-1}{x-1} = \frac{\cancel{(x-1)}(x+1)}{\cancel{x-1}}$$

$$= \begin{cases} 2 & x < -2 \\ x+1, & x \geq -2, \neq 1 \\ -3, & x = 1 \end{cases}$$