3.1 Polynomial Functions & Modeling

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots + a_2 x^2 + a_1 x + a_0$$

$a_n, a_{n-1}, \ldots, a_1, a_0$'s = real #'ed coefficients

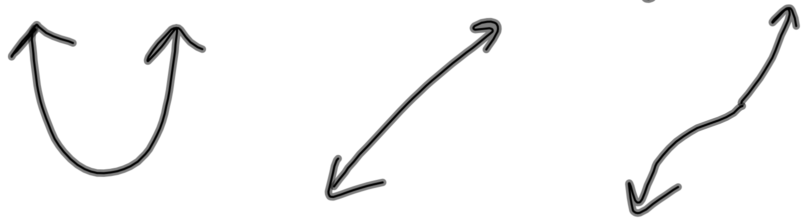
n = degree

a_n = leading coefficient

a_0 = constant term

Is degree even or odd?

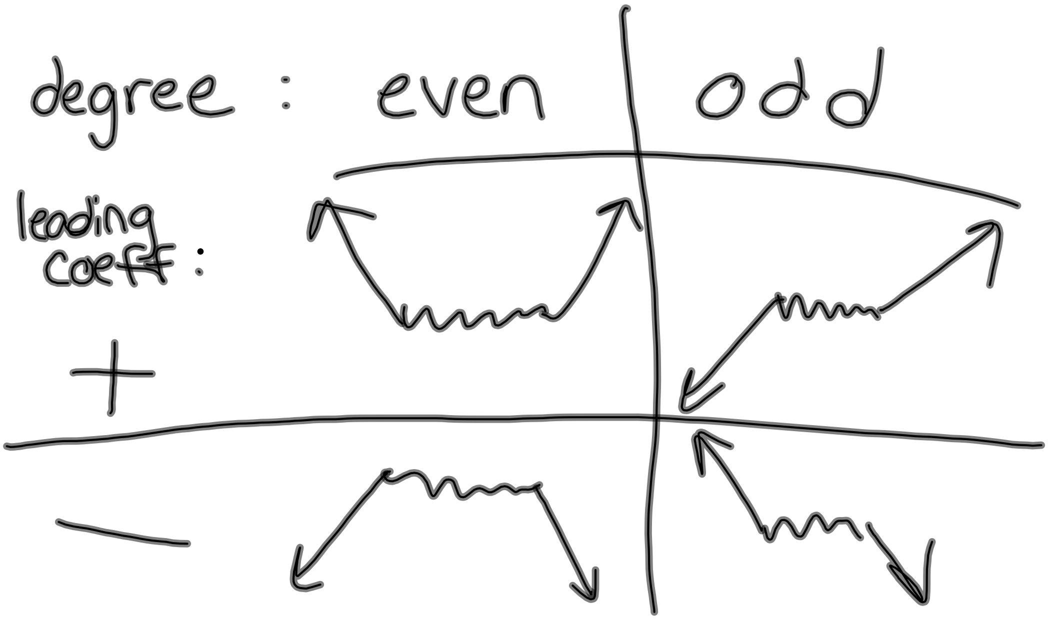
$y = x^2$ $\quad$ $y = x$ $\quad$ $y = x^3$



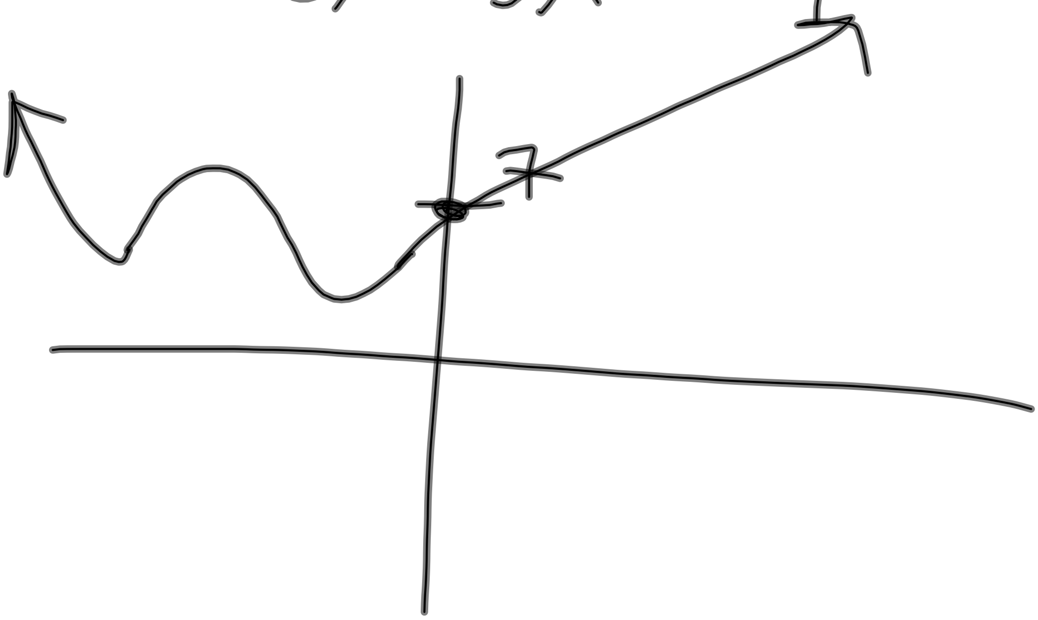
as $x \to \pm\infty$, $y \to +\infty$

as $x \to +\infty$, $y \to +\infty$; as $x \to -\infty$, $y \to -\infty$

If leading coeff. is negative, vertical flip



$f(x) = 5x^4 - 3x^2 + 7$



degree determines # of zeros:

The fundamental Theorem of Algebra

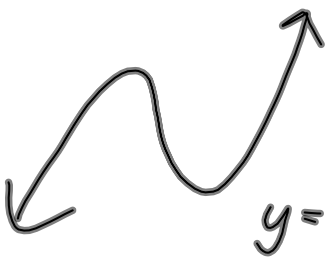
An n^{th} degree polynomial has n zeros

$$f(x) = (x-b_1)(x-b_2)\cdots(x-b_n)$$

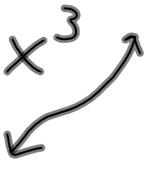
and can be written as the product of n linear factors

The graph of an n^{th} degree polynomial has at most $n-1$ turning points.

cubic:



$y=x^3$



6^{th} deg:

$$f(x) = -2x^5 - x^3$$

$-2x^5 - x^3 = 0$

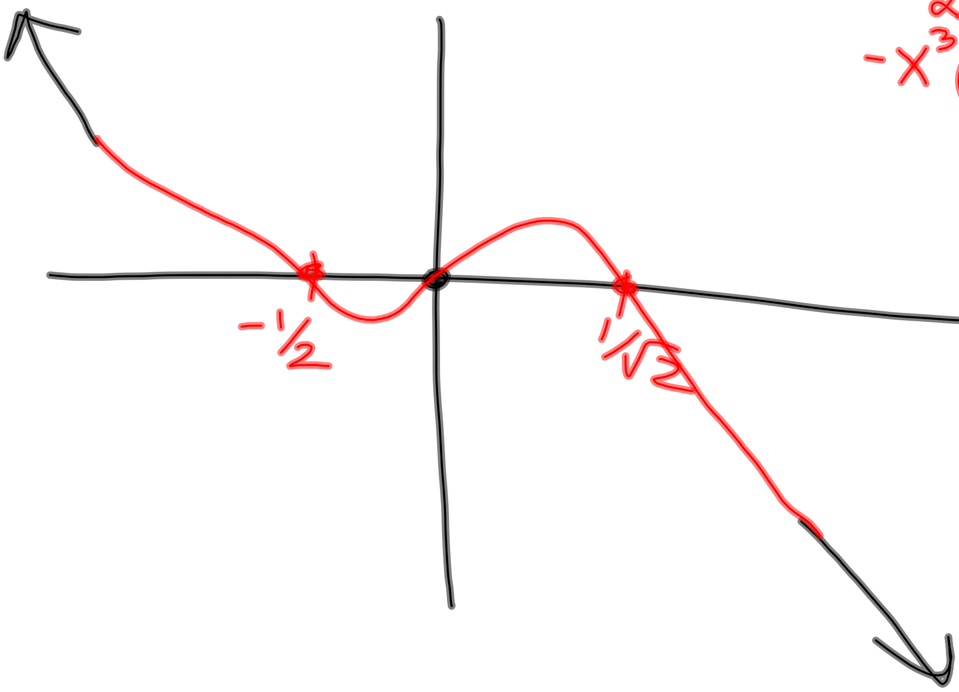
$-x^3(2x^2 - 1) = 0$

$2x^2 - 1 = 0$

$2x^2 = 1$

$x^2 = \frac{1}{2}$

$x = \pm \frac{1}{\sqrt{2}}$



$$y = -x^4 + 7x^3 - 6x^2$$

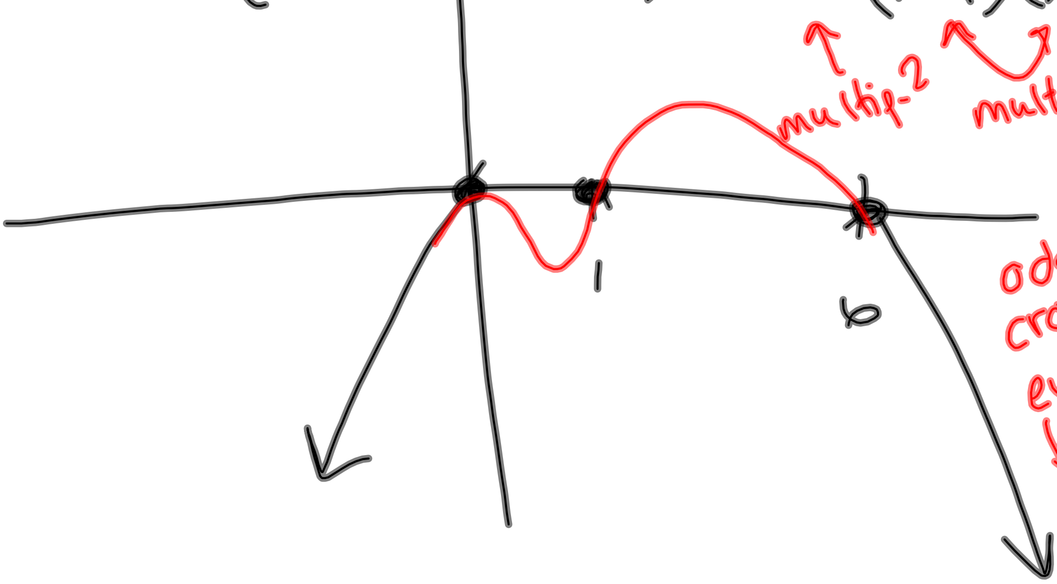
$$= -x^2(x^2 - 7x + 6) = -x^2(x-6)^1(x-1)^1$$

multip. 2

multiplicity 1

odd mult: cross thru

even mult: bounce off



$y = (x-2)^4(x+3)^3$ zeros 2 -3

lead term: x^7 mult 4 mult 3