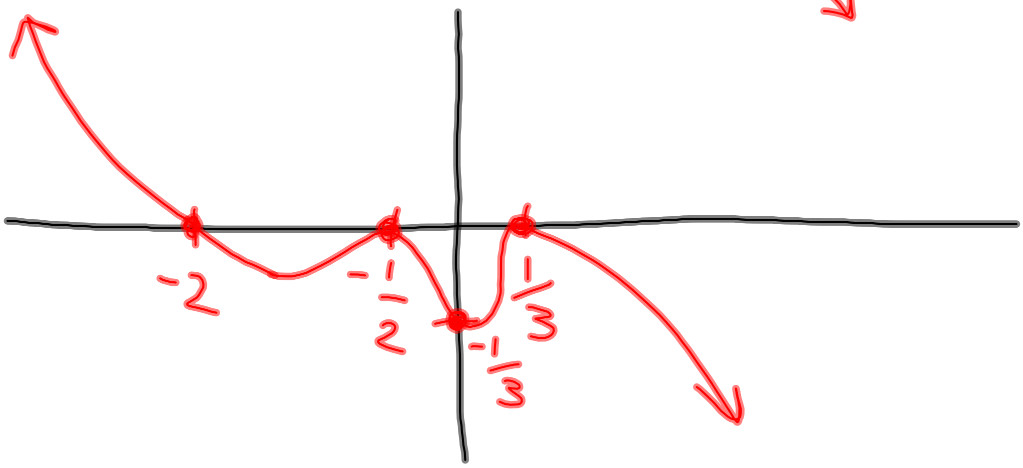
Graph: $f(x) = -6\left(x+\frac{1}{2}\right)^2\left(x-\frac{1}{3}\right)^2(x+2)$

zeros:	$-1/2$	$1/3$	-2
multiplicity:	2 even	2 even	1 odd

y-intercept: $-6\left(\frac{1}{2}\right)^2\left(-\frac{1}{3}\right)^2(2) = -\cancel{12}^{\,3}\cdot\frac{1}{\cancel{4}}\cdot\frac{1}{9} = -\frac{1}{3}$ $\quad\left(0, -\frac{1}{3}\right)$

lead term: $-6x^2x^2x = -6x^5$



3.3 Zeros of Polynomials & Polynomial Division

$f(x) = (x-a)(x-b)(x-c)$

$\Rightarrow$ zeros are $a, b, \& c$.

$12 \div 3 = 4 \Rightarrow 12 = 3\cdot 4$

Long Division

3.3 #6

$P(x) = 2x^3 - 3x^2 + x - 1$

$d(x) = x - 3$

$\frac{P(x)}{d(x)} = P(x) \div d(x)$

$$\begin{array}{r} 2x^2 + 3x + 10 \\ x-3 \overline{\smash{)}\, 2x^3 - 3x^2 + x - 1} \\ -(2x^3 - 6x^2) \\ \hline 3x^2 + x - 1 \\ -(3x^2 - 9x) \\ \hline 10x - 1 \\ -(10x - 30) \\ \hline 29 \end{array}$$

$P(3) = 29$

$\Rightarrow \frac{2x^3 - 3x^2 + x - 1}{x - 3} = 2x^2 + 3x + 10 + \frac{29}{x-3}$

$\left(\frac{3}{2} = 1 + \frac{1}{2}\right)$

$\Rightarrow 2x^3 - 3x^2 + x - 1 = (2x^2 + 3x + 10)(x - 3) + 29$

Quotient: $2x^2 + 3x + 10$; Remainder: 29

8. $P(x) = x^3 - 9x^2 + 15x + 25$

$d(x) = x - 5$

$$\begin{array}{r} x^2 - 4x - 5 \\ x-5 \overline{\smash{)}\, x^3 - 9x^2 + 15x + 25} \\ -(x^3 - 5x^2) \\ \hline -4x^2 + 15x + 25 \\ -(-4x^2 + 20x) \\ \hline -5x + 25 \\ -(-5x + 25) \\ \hline 0 \end{array}$$

$P(5) = 0$

$x^3 - 9x^2 + 15x + 25 = (x - 5)(x^2 - 4x - 5)$

Synthetic Division

12. $\underbrace{(x^3-7x^2+13x+3)}_{P(x)} \div (x-2)$

$$\begin{array}{r|rrrr}
2 & 1 & -7 & 13 & 3 \\
  &   & 2 & -10 & 6 \\
\hline
  & 1 & -5 & 3 & \boxed{9} \\
  & \uparrow & \uparrow & \uparrow & \uparrow \\
  & x^2\text{-coeff} & x\text{-coeff} & \text{constant} & \text{remainder}
\end{array}$$

Quotient: x^2-5x+3; R: 9

$x^3-7x^2+13x+3=(x-2)(x^2-5x+3)+9$

$P(2)=9$

18. $\underbrace{(x^7-x^6+x^5-x^4+2)}_{P(x)} \div (x+1)$

$$\begin{array}{r|rrrrrrrr}
-1 & 1 & -1 & 1 & -1 & 0 & 0 & 0 & 2 \\
   &   & -1 & 2 & -3 & 4 & -4 & 4 & -4 \\
\hline
   & 1 & -2 & 3 & -4 & 4 & -4 & 4 & \boxed{-2} \\
   & x^6 & x^5 & x^4 & x^3 & x^2 & x & a_0 &
\end{array}$$

$x^7-x^6+x^5-x^4+2=$

$(x+1)(x^6-2x^5+3x^4-4x^3+4x^2-4x+4)-2$

$P(-1)=-2$

20. $\underbrace{(x^5+32)}_{P(x)} \div (x+2)$

$$\begin{array}{r|rrrrrr} -2 & 1 & 0 & 0 & 0 & 0 & 32 \\ & & -2 & 4 & -8 & 16 & -32 \\ \hline & 1 & -2 & 4 & -8 & 16 & \boxed{0} \\ & x^4 & x^3 & x^2 & x & a_0 & \end{array}$$

$x^5+32 = (x+2)(x^4-2x^3+4x^2-8x+16)$

$\Rightarrow P(2) = 0$

32. $f(x) = 3x^3 + 11x^2 - 2x + 8$

are the #'s -4 & 2 zeros of the function?

$$\begin{array}{r|rrrr} -4 & 3 & 11 & -2 & 8 \\ & & -12 & 4 & -8 \\ \hline & 3 & -1 & 2 & \boxed{0} \end{array}$$

Yes

$$\begin{array}{r|rrrr} 2 & 3 & 11 & -2 & 8 \\ & & 6 & 34 & 64 \\ \hline & 3 & 17 & 32 & \boxed{72} \end{array}$$

No

46. $f(x) = x^4 - 4x^3 - 7x^2 + 34x - 24$

Factor the polynomial and solve $f(x) = 0$.

Not so hard to see that $f(1) = 0$.

$$\begin{array}{r|rrrr|r} 1 & 1 & -4 & -7 & 34 & -24 \\ & & 1 & -3 & -10 & 24 \\ \hline & 1 & -3 & -10 & 24 & \boxed{0} \\ & x^3 & x^2 & x & a_0 & \end{array}$$

$f(x) = (x-1)(x^3 - 3x^2 - 10x + 24)$

2 is also a zero.

$$\begin{array}{r|rrr|r} 2 & 1 & -3 & -10 & 24 \\ & & 2 & -2 & -24 \\ \hline & 1 & -1 & -12 & \boxed{0} \\ & x^2 & x & a_0 & \end{array}$$

$f(x) = (x-1)(x-2)(x^2 - x - 12)$

$$\boxed{f(x) = (x-1)(x-2)(x-4)(x+3)}$$

$\Rightarrow$ zeros: $1, 2, 4, -3$

3.3

#9, 13, 19, 21, 23