3.1 # 8-14, 23-32 all!

3.2 # 16, 17, 21, 22, 24, 25, 27, 28

3.3 # 9, 13, 19, 21, 23

Review:

Determine the domain:

$f(x) = \frac{5}{x-2}$; $g(x) = \sqrt{x+3}$; $(f\circ g)(x)$

$x-2 \neq 0$

$x \neq 2$

$\{x \mid x \neq 2\}$

$(-\infty, 2) \cup (2, \infty)$

$x+3 \geq 0$

$x \geq -3$

$\{x \mid x \geq -3\}$

$[-3, \infty)$

$$(f\circ g)(x) = \frac{5}{\sqrt{x+3}-2}$$

$x+3 \geq 0$

$x \geq -3$

$\sqrt{x+3} - 2 \neq 0$

$\sqrt{x+3} \neq 2$

$x+3 \neq 4$

$x \neq 1$

$\{x \mid x \geq -3\} \cap \{x \mid x \neq 1\}$

$\boxed{[-3, 1) \cup (1, \infty)} = \{x \mid -3 \leq x < 1 \text{ or } x > 1\}$

3.3

38. Determine if i or $-i$ are zeros of

$$f(x) = x^3 + 2x^2 + x + 2$$

$$\begin{array}{r|rrrr} i & 1 & 2 & 1 & 2 \\ & & i & \begin{array}{c} i(2+i) \\ 2i+i^2 \\ 2i-1 \end{array} & \begin{array}{c} i(2i) \\ -2 \end{array} \\ \hline & 1 & 2+i & 2i & \boxed{0} \end{array}$$

$\Rightarrow i$ is a zero

$$\begin{array}{r|rrrr} -i & 1 & 2 & 1 & 2 \\ & & -i & \begin{array}{c} -i(2-i) \\ -2i-1 \end{array} & -2 \\ \hline & 1 & 2-i & -2i & \boxed{0} \end{array}$$

$\Rightarrow -i$ is a zero

$-2i + i^2$

$(i)^2 = (\sqrt{-1})^2$

$i^2 = -1$

If the discriminant $b^2-4ac<0$,
$\Rightarrow$ 2 complex conjugate zeros

$\Rightarrow$ if i is a zero, so is $-i$

If $a+bi$ is a zero, so is $a-bi$

If $f(x)$ has $a+bi$ & $a-bi$ as its only 2 zeros,

$$f(x)=[x-(a+bi)][x-(a-bi)]$$
$$=(x-a-bi)(x-a+bi)$$

$$f(x)=x^3+2x^2+x+2$$

knowing that i & $-i$ are zeros of f

$x^2=-1 \Rightarrow x^2+1=0$
$x=\pm i$

$$\begin{array}{r|l} & x+2 \\ \hline x^2+1 & x^3+2x^2+x+2 \\ & -(x^3+x) \\ \hline & 2x^2+2 \\ & -(2x^2+2) \\ \hline & 0 \end{array}$$

$\Rightarrow f(x)=(x^2+1)(x+2)$
$f(x)=(x+i)(x-i)(x+2)$
zeros are $-i, i$ & -2

$a+\sqrt{b}$ & $a-\sqrt{b}$
also come in pairs

If 2, $\sqrt{3}$, & $2-i$ are zeros of $p(x)$,

$$p(x) = (x-2)(x-\sqrt{3})(x-(2-i))\cdot(x+\sqrt{3})(x-(2+i))$$

Rational Zeros Theorem

If $p(x) = a_n x^n + \ldots + a_1 x + a_0$

The only possible rational zeros are of the form

$$\frac{\pm \text{factors of } a_0}{\text{factors of } a_n}$$

$$p(x) = 4x^3 - 7x^2 + 21x - 5$$

possible rational zeros:

$$\pm \frac{\text{factors of } 5}{\text{factors of } 4} = \pm \frac{1, 5}{1, 2, 4}$$

$$= \pm 1, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm 5, \pm \frac{5}{2}, \pm \frac{5}{4}$$

3.4 ~~74~~. $f(x) = 2x^3 + 3x^2 + 2x + 3$

possible rational zeros:

$$\pm \frac{\text{factors of } 3}{\text{factors of } 2} = \frac{\pm 1, \pm 3}{\pm 1, \pm 2} = \pm 1, \pm \frac{1}{2}, \pm 3, \pm \frac{3}{2}$$

$\cancel{f(-3)} =$

$\cancel{f(\tfrac{1}{2})} =$

$f(-\tfrac{3}{2}) = 0$

$$\begin{array}{r|rrrr} -\frac{3}{2} & 2 & 3 & 2 & 3 \\ & & -3 & 0 & -3 \\ \hline & 2 & 0 & 2 & \boxed{0} \end{array}$$

$$\begin{array}{r|rrrr} -3 & 2 & 3 & 2 & 3 \\ & & -6 & +9 & \cancel{-21}\ {-33} \\ \hline & 2 & -3 & \cancel{-7}\ 11 & \cancel{24}\ {-30} \end{array}
\qquad
\begin{array}{r|rrrr} -\frac{1}{2} & 2 & 3 & 2 & 3 \\ & & -1 & -1 & -\frac{1}{2} \\ \hline & 2 & 2 & 1 & \cancel{} \end{array}$$

$$\Rightarrow f(x) = (x + \tfrac{3}{2})(2x^2 + 2)$$

$$= 2(x + \tfrac{3}{2})(x^2 + 1)$$

$$\boxed{f(x) = 2(x + \tfrac{3}{2})(x + i)(x - i)}$$

zeros: $-3/2, -i, i$

Descartes' Rule of Signs

If $P(x)$ is written in descending order w/ real # coefficients and a nonzero constant term,

The # of positive real zeros is either
- the # of sign changes of $P(x)$
- or less than that # by a positive even integer

The # of negative real zeros is either
- the # of sign changes of $P(-x)$
- or less than that # by a positive even integer

HW

3.3 #35

3.4 #17-31 odd, 51, 53