3.1 # 8-14, 23-32 all!
3.2 # 16, 17, 21, 22, 24, 25, 27, 28

3.3
9, 13, 19, 21, 23

Review:

Determine the domain:

$f(x)=\frac{5}{x-2}$; $g(x)=\sqrt{x+3}$; $(f\circ g)(x)$

$x-2\neq 0$
$x\neq 2$
$\{x \mid x\neq 2\}$
$(-\infty,2)\cup(2,\infty)$

$x+3\geq 0$
$x\geq -3$
$\{x \mid x\geq -3\}$
$[-3,\infty)$

$\frac{5}{\sqrt{x+3}-2}$

2 conditions we must satisfy

$x+3\geq 0$
$\boxed{x\geq -3}$

$\sqrt{x+3}-2\neq 0$
$\sqrt{x+3}\neq 2$
$x+3\neq 4$
$\boxed{x\neq 1}$

$\{x \mid x\geq -3\}\cap\{x \mid x\neq 1\}$

$\boxed{[-3,1)\cup(1,\infty)}$

$\{x \mid -3\leq x<1 \text{ or } x>1\}$

3.3

38. Determine if i or $-i$ are zeros of

$f(x)=x^3+2x^2+x+2$

$$\begin{array}{r|rrrr} i & 1 & 2 & 1 & 2 \\ & & i & i(2+i) = 2i+i^2 = 2i-1 & i(2i)=2i^2 = -2 \\ \hline & 1 & 2+i & 2i & \boxed{0} \end{array}$$

$\Rightarrow i$ is a zero

$$\begin{array}{r|rrrr} -i & 1 & 2 & 1 & 2 \\ & & -i & -2i-1 & -2 \\ \hline & 1 & 2-i & -2i & \boxed{0} \end{array}$$

$\Rightarrow -i$ is a zero

If the discriminant $b^2-4ac<0$,
$\Rightarrow$ 2 complex conjugate zeros

$\Rightarrow$ if i is a zero, so is $-i$

If $a+bi$ is a zero, so is $a-bi$

If $f(x)$ has $a+bi$ & $a-bi$ as its only 2 zeros,

$$f(x)=[x-(a+bi)][x-(a-bi)]$$
$$=(x-a-bi)(x-a+bi)$$

$$f(x)=x^3+2x^2+x+2$$

knowing that i & $-i$ are zeros of f

$x^2=-1$ $\quad$ $x^2+1=0$
$x=\pm i$

$$\begin{array}{r|l} & x+2 \\ \hline x^2+1 & x^3+2x^2+x+2 \\ & -(x^3 \quad\quad +x) \\ \hline & 2x^2+2 \\ & -(2x^2+2) \\ \hline & 0 \end{array}$$

$$f(x)=(x+i)(x-i)(x+2)$$
zeros: $\pm i, -2$

$a+\sqrt{b}$ & $a-\sqrt{b}$
also come in pairs

If 2, $\sqrt{3}$, & $2-i$ are zeros of $p(x)$,

$$p(x) = (x-2)(x-\sqrt{3})(x-(2-i))\cdot(x+\sqrt{3})(x-(2+i))$$

Rational Zeros Theorem

If $p(x) = a_n x^n + \dots + a_1 x + a_0$

The only possible rational zeros are of the form

$$\frac{\pm \text{factors of } a_0}{\text{factors of } a_n}$$

$$p(x) = 4x^3 - 7x^2 + 21x - 5$$

possible rational zeros:

$$\frac{\text{factors of } 5}{\text{factors of } 4} = \frac{\pm 1, 5}{\pm 1, 2, 4}$$

$$= \pm 1, \pm \frac{1}{2}, \pm \frac{5}{2}, \pm \frac{1}{4}, \pm 5, \pm \frac{5}{4}$$

$\frac{3.4}{74}$. $f(x) = 2x^3 + 3x^2 + 2x + 3$

possible rational zeros

$$\frac{\text{factors of } 3}{\text{factors of } 2} : \frac{\pm 1, 3}{\pm 1, 2} = \cancel{\pm 1}, \pm \frac{1}{2}, \cancel{\pm 3}, \pm \frac{3}{2}$$

$$2\left(\frac{-1}{2}\right)^3 + 3\left(\frac{-1}{2}\right)^2 + 2\left(\frac{-1}{2}\right) + 3$$

$$2\left(\frac{-3}{2}\right)^3 + 3\left(\frac{-3}{2}\right)^2 + 2\left(\frac{-3}{2}\right) + 3$$

$$\begin{array}{r|rrrr} -\frac{1}{2} & 2 & 3 & 2 & 3 \\ & & -1 & -1 & -\frac{1}{2} \\ \hline & 2 & 2 & 1 & \times \end{array} \qquad \begin{array}{r|rrrr} -\frac{3}{2} & 2 & 3 & 2 & 3 \\ & & -3 & 0 & -3 \\ \hline & 2 & 0 & 2 & \boxed{0} \end{array}$$

$$2x^3 + 3x^2 + 2x + 3 = \left(x + \frac{3}{2}\right)\left(2x^2 + 2\right)$$

$$2(x^2+1)$$

$$f(x) = 2\left(x + \frac{3}{2}\right)(x - i)(x + i)$$

Descartes' Rule of Signs

If $P(x)$ is written in descending order w/ real # coefficients and a nonzero constant term,

The # of positive real zeros is either
- the # of sign changes of $P(x)$
- or less than that # by a positive even integer

The # of negative real zeros is either
- the # of sign changes of $P(-x)$
- or less than that # by a positive even integer

HW

3.3 #35

3.4 #17-31 odd, 51, 53