For the polynomial

$$p(x) = -2\left(x-\frac{1}{3}\right)^2(x-3)^3(x+2)^2\left(x+\frac{1}{2}\right)^3$$

Identify:

1. The zeros and their multiplicities: $\frac{1}{3}$ m2, 3 m3, -2 m2, $-\frac{1}{2}$ m3

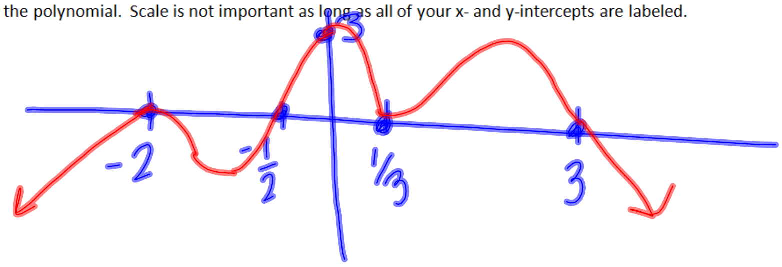
2. The y-intercept: $(0,3)$

$$-2\left(0-\tfrac{1}{3}\right)^2(0-3)^3(0+2)^2\left(0+\tfrac{1}{2}\right)^3 = -2\left(-\tfrac{1}{3}\right)^2(-3)^3(2)^2\left(\tfrac{1}{2}\right)^3 = \frac{-2}{1}\cdot\frac{1}{\cancel{9}}\cdot\frac{\cancel{-27}}{1}\cdot\frac{\cancel{4}}{1}\cdot\frac{1}{\cancel{8}}$$

3. The lead term and a sketch of what that implies for the end behavior of the graph:

$$-2x^2x^3x^2x^3 = -2x^{10}$$

4. Graph the polynomial. Scale is not important as long as all of your x- and y-intercepts are labeled.



Bonus: For $f(x) = \frac{1}{x}$ and $g(x) = \sqrt{x-1}$, write and find the domain of $(f\circ g)(x)$.

$(f\circ g)(x) = \dfrac{1}{\sqrt{x-1}}$ $\quad x-1>0$

$\{x \mid x>1\}$ $\quad (1,\infty)$

3.4

$a+bi,\ a-bi$

$a+b\sqrt{c},\ a-b\sqrt{c}$

17. $-1, \sqrt{3}, \frac{11}{3}$

$\Rightarrow -\sqrt{3}$

21. $3i, 0, -5$

$\Rightarrow -3i$

19. $-i, 2-\sqrt{5} \Rightarrow i, 2+\sqrt{5}$

Descartes' Rule of Signs

If $P(x)$ is written in descending order w/ real # coefficients and a nonzero constant term,

The # of positive real zeros is either
- the # of sign changes of $P(x)$
- or less than that # by a positive even integer

The # of negative real zeros is either
- the # of sign changes of $P(-x)$
- or less than that # by a positive even integer

3.4

84. $H(t) = 5t^{12} - 7t^4 + 3t^2 + t + 1$

$+ \quad - \quad + \quad + \quad +$

2 sign changes

$\Rightarrow$ either 2 positive real zeros or 0 positive real zeros

$H(-t) = 5(-t)^{12} - 7(-t)^4 + 3(-t)^2 + (-t) + 1$
$= 5t^{12} - 7t^4 + 3t^2 - t + 1$

$+ \quad - \quad + \quad - \quad +$

4 sign changes

$\Rightarrow$ 4 or 2 or 0 negative real zeros

80. $g(x) = 5x^6 - 3x^3 + x^2 - x$

$$= x(\underbrace{5x^5 - 3x^2 + x - 1}_{f(x)})$$

$+ \quad - \quad + \quad -$

3 sign changes

$\Rightarrow$ either 3 or 1 positive real zeros

$$f(-x) = 5(-x)^5 - 3(-x)^2 + (-x) - 1$$

$$= -5x^5 - 3x^2 - x - 1$$

$- \quad - \quad - \quad -$

0 sign changes

$\Rightarrow$ no negative real zeros

86. $g(z) = -z^{10} + 8z^7 + z^3 + 6z - 1$

$- \quad + \quad + \quad + \quad -$

2 sign changes

$\Rightarrow$ 2 or 0 positive real zeros

$$g(-z) = -(-z)^{10} + 8(-z)^7 + (-z)^3 + 6(-z) - 1$$

$$= -z^{10} - 8z^7 - z^3 - 6z - 1$$

$- \quad - \quad - \quad - \quad -$

0 sign changes $\Rightarrow$ no negative real zeros

3.4

#55-69 odd ; #95-98 all

#79, 89, 93