

For the polynomial

$$p(x) = -2\left(x - \frac{1}{3}\right)^2 (x - 3)^3 (x + 2)^2 \left(x + \frac{1}{2}\right)^3$$

Identify:

1. The zeros and their multiplicities:

$$\begin{matrix} \frac{1}{3} & 3 & -2 & -\frac{1}{2} \\ m_2 & m_3 & m_2 & m_3 \end{matrix}$$

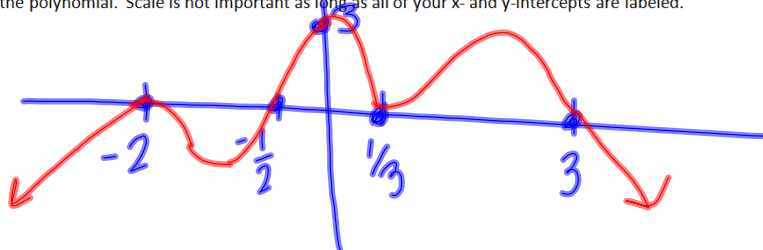
2. The y-intercept:

$$2\left(0 - \frac{1}{3}\right)^2 (0 - 3)^3 (0 + 2)^2 \left(0 + \frac{1}{2}\right)^3 = -2\left(\frac{1}{3}\right)^2 (-3)^3 (2)^2 \left(\frac{1}{2}\right)^3 = \frac{2 \cdot 1 \cdot 27 \cdot 4 \cdot 1}{1 \cdot 9 \cdot 1 \cdot 1 \cdot 8} = (0, 3)$$

3. The lead term and a sketch of what that implies for the end behavior of the graph:

$$-2x^2 x^3 x^2 x^3 = -2x^{10}$$

4. Graph the polynomial. Scale is not important as long as all of your x- and y-intercepts are labeled.



Bonus: For $f(x) = \frac{1}{x}$ and $g(x) = \sqrt{x - 1}$, write and find the domain of $(f \circ g)(x)$.

$$(f \circ g)(x) = \frac{1}{\sqrt{x-1}} \quad \left\{ \begin{array}{l} x-1 > 0 \\ x > 1 \end{array} \right\} \quad (1, \infty)$$

3.4

17. $-1, \sqrt{3}, \frac{11}{3}$

$$\Rightarrow -\sqrt{3}$$

$$a+bi, a-bi$$

$$a+b\sqrt{c}, a-b\sqrt{c}$$

21. $3i, 0, -5$

$$\Rightarrow -3i$$

19. $-i, 2-\sqrt{5} \Rightarrow i, 2+\sqrt{5}$

Descartes' Rule of Signs

If $P(x)$ is written in descending order w/ real # coefficients and a non-zero constant term,

The # of positive real zeros is either

- the # of sign changes of $P(x)$
- or less than that # by a positive even integer

The # of negative real zeros is either

- the # of sign changes of $P(-x)$
- or less than that # by a positive even integer

3.4

$$84. \quad H(t) = 5t^{12} - 7t^4 + 3t^2 + t + 1$$

+ - + + +

2 sign changes

⇒ either 2 positive real zeros

0 positive real zeros

$$H(-t) = 5(-t)^{12} - 7(-t)^4 + 3(-t)^2 + (-t) + 1$$

$$= 5t^{12} - 7t^4 + 3t^2 - t + 1$$

+ - + - +

4 sign changes

⇒ either 4 or 2 or 0

negative real zeros

$$80. \quad g(x) = 5x^6 - 3x^3 + x^2 - x$$

$$= x(5x^5 - 3x^2 + x - 1)$$

+ - + -
- + -

3 sign changes \Rightarrow

either 3 or 1 positive real zeros

$$g(-x) = 5(-x)^5 - 3(-x)^2 + (-x) - 1$$

$$= -5x^5 - 3x^2 - x - 1$$

- - - -

0 sign changes \Rightarrow

0 negative real zeros

$$86. \quad g(z) = -z^{10} + 8z^7 + z^3 + 6z - 1$$

- + + + -
- +

2 sign changes

\Rightarrow 2 or 0 positive real zeros

$$g(-z) = -(-z)^{10} + 8(-z)^7 + (-z)^3 + 6(-z) - 1$$

$$= -z^{10} - 8z^7 - z^3 - 6z - 1$$

- - - - -

0 sign changes

\Rightarrow 0 negative real zeros

3.4

55-69 odd ; # 95-98 all

79, 89, 93