Factor:

$$f(x) = \underbrace{x^3+3x^2}\;\underbrace{-2x-6}$$

$$= x^2(x+3) - 2(x+3)$$

$$f(x) = (x+3)\underbrace{(x^2-2)}$$

$$\boxed{f(x) = (x+3)(x+\sqrt{2})(x-\sqrt{2})}$$

$$x^2 - 2 = 0$$
$$x^2 = 2$$
$$x = \pm\sqrt{2}$$

63. $f(x) = x^4 - 3x^3 - 20x^2 - 24x - 8$

$\underline{-1}|$ 1 -3 -20 -24 -8

$\underline{-2}|$ 1 -4 -16 -8

96. $f(x)=3x^3-4x^2-5x+2$

$-1 \rfloor \quad 3 \quad -4 \quad -5 \quad 2$

98. $f(x)=3x^4-37x^2+9$

$3x^4$ ↖ ↗

y-int $(0,9)$

Let $u=x^2$

$f(u)=3u^2-37u+9$

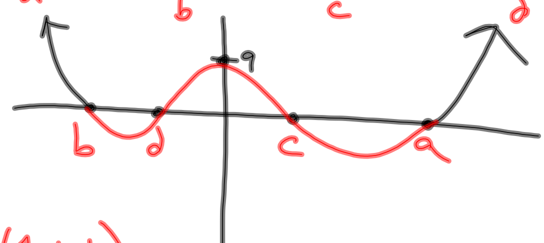
$$u=\frac{-(-37)\pm\sqrt{(-37)^2-4(3)(9)}}{2(3)}$$

$$x^2=u=\frac{37\pm\sqrt{1261}}{6}$$

$$x=\pm\sqrt{\frac{37\pm\sqrt{1261}}{6}}$$

42
$$\begin{array}{r} 37 \\ \times\, 37 \\ \hline 259 \\ 1110 \\ \hline 1369 \\ -108 \\ \hline 1261 \end{array}$$

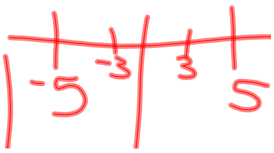
$$\underset{a}{\sqrt{\frac{37+\sqrt{1261}}{6}}},\ \underset{b}{-\sqrt{\frac{37+\sqrt{1261}}{6}}},\ \underset{c}{\sqrt{\frac{37-\sqrt{1261}}{6}}},\ \underset{d}{-\sqrt{\frac{37-\sqrt{1261}}{6}}}$$



$\pm(4\pm1)$

$|4+1| > |4-1|$

5 3



3.5 Rational Functions

$f(x) = \frac{p(x)}{q(x)}$ fractions made of polynomials!

y-int: $(0, f(0))$ [plug 0 in for x]
unless $f(0)$ is undefined, in which case there is no y-intercept

x-int/zeros: occur when numerator = 0, ie. solutions to $p(x)=0$.

when denominator is 0, $f(x)$ is undefined (i.e. $q(x)=0$)

If a particular x-value makes both numerator & denominator 0, that factor will cancel and we will have a hole in the graph

If an x-value makes only the denominator equal to 0, the graph will have a vertical asymptote @ that x-value

End behavior (what happens to $f(x)$ as $x \to \pm\infty$)

Look @ ratio of lead terms.

- If denominator has a higher degree, horizontal asymptote $y=0$.
  ($f(x) \to 0$ as $x \to \pm\infty$)

E.g. $f(x) = \frac{-3x^5 - 2x^2 + 3}{5x^9 + 2x}$

$$\frac{-3x^5}{5x^9} = \frac{-3}{5x^4} \to 0 \text{ as } x \to \pm\infty$$

- If numerator & denominator have the same degree, then horizontal asymptote is y = ratio of leading coefficients

e.g. $f(x) = \dfrac{5x^2-4}{10x^2+3x}$ H.A. is $y=\frac{1}{2}$

$\dfrac{5x^2}{10x^2} = \dfrac{5}{10} = \dfrac{1}{2}$ $f(x) \to \dfrac{1}{2}$ as $x \to \pm\infty$

- If numerator has a larger degree, perform long division.

If deg is 1 higher $\longrightarrow$ linear "oblique" asymptote

If deg is 2 higher $\longrightarrow$ parabolic asymptote

3 $\rightarrow$ cubic asymptote

. . .

$$f(x)=\frac{x^2-7x+6}{x+5}=\frac{(x-6)(x-1)}{x+5}$$

zeros: 1, 6

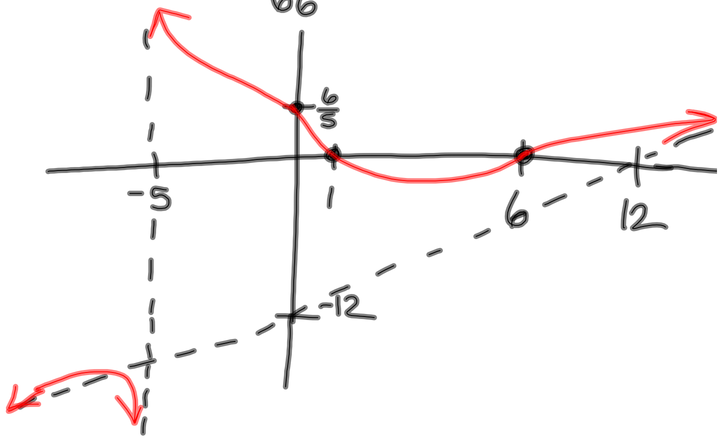
y-int: $(0,\frac{6}{5})$

vertical asymptote: $x=-5$

$$\begin{array}{r} x-12 \\ x+5\overline{)\,x^2-7x+6} \\ -(x^2+5x) \\ \hline -12x+6 \\ -(-12x-60) \\ \hline 66 \end{array}$$

quotient: $x-12+\frac{66}{x+5}$ $\frac{66}{x+5}\to 0$

oblique asymptote: $y=x-12$



$$f(x)=\frac{2x(x+3)(x-1)}{(x-4)(x+1)(x+6)}$$

$f(-2)=\frac{-+-}{--+}>0$

$f(2)=\frac{+++}{-++}<0$

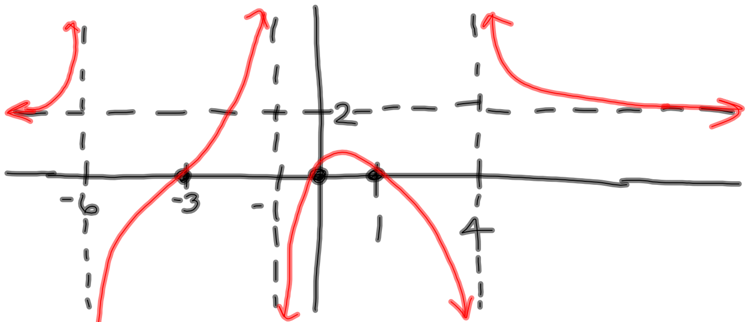
zeros: 0, -3, 1 ; y-int: $(0,0)$

vertical asymptotes: $x=4$, $x=-1$, $x=-6$

end behavior:

$$\frac{2x\cdot x\cdot x}{x\cdot x\cdot x}=\frac{2x^3}{x^3}=2$$

horizontal asymptote $y=2$



as $x\to-\infty$, $f(x)\to 2$; as $x\to\infty$, $f(x)\to 2$

as $x\to-6^-$, $f(x)\to\infty$; as $x\to-6^+$, $f(x)\to-\infty$

from the left

as $x\to-1^-$, $f(x)\to\infty$; as $x\to-1^+$, $f(x)\to-\infty$

as $x\to4^-$, $f(x)\to-\infty$; as $x\to4^+$, $f(x)\to\infty$

HW:

3.5 # 7-25 odd