Factor:

$f(x) = \underbrace{x^3 + 3x^2}\ \underbrace{-2x - 6}$

$= x^2(x+3) - 2(x+3)$

$= (x+3)(x^2-2)$

$x^2 - 2 = 0$

$x^2 = 2$

$x = \pm\sqrt{2}$

$f(x) = (x+3)(x-\sqrt{2})(x+\sqrt{2})$

Zeros: $-3, \sqrt{2}, -\sqrt{2}$

63. $f(x) = x^4 - 3x^3 - 20x^2 - 24x - 8$

$$\begin{array}{r|rrrrr} -1 & 1 & -3 & -20 & -24 & -8 \\ & & -1 & 4 & 16 & 8 \\ \hline & 1x^3 & -4x^2 & -16x & -8 & \boxed{0} \end{array}$$

$f(x) = (x+1)(x^3 \underbrace{-4x^2 - 16x} - 8)$

$$\begin{array}{r|rrrr} -2 & 1 & -4 & -16 & -8 \\ & & -2 & 12 & 8 \\ \hline & 1x^2 & -6x & -4 & \boxed{0} \end{array}$$

$f(x) = (x+1)(x+2)(x^2 - 6x - 4)$

$x = \dfrac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-4)}}{2(1)}$

$= \dfrac{6 \pm \sqrt{36+16}}{2} = \dfrac{6 \pm \sqrt{52}}{2}$

$= \dfrac{6 \pm 2\sqrt{13}}{2} = 3 \pm \sqrt{13}$

$f(x) = (x+1)(x+2)(x-(3+\sqrt{13}))(x-(3-\sqrt{13}))$

96. $f(x)=3x^3-4x^2-5x+2$

$$\begin{array}{r|rrrr} -1 & 3 & -4 & -5 & 2 \\ & & -3 & 7 & -2 \\ \hline & 3 & -7 & 2 & \boxed{0} \end{array}$$

$f(x)=(x+1)(3x^2-7x+2)$

$3x^2-6x-x+2$

$3x(x-2)-1(x-2)$

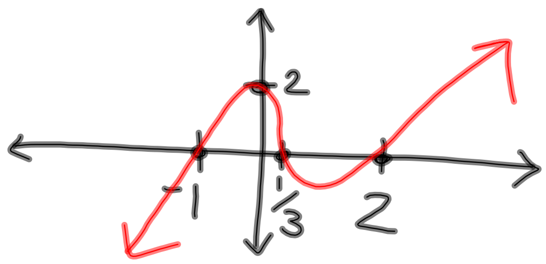
$(x-2)(3x-1)$

$f(x)=(x+1)^1(x-2)^1(3x-1)^1$

lead term: $3x^3$

y-int: $(0,2)$

zeros: $-1, 2, \frac{1}{3}$ all mult 1



3.5 Rational Functions

$f(x)=\dfrac{p(x)}{q(x)}$ fractions made of polynomials!

y-int: $(0, f(0))$ [plug 0 in for x]
unless $f(0)$ is undefined, in which case there is no y-interrept

x-int/zeros: occur when numerator $=0$, ie. solutions to $p(x)=0$.

when denominator is 0, $f(x)$ is undefined (i.e. $q(x)=0$)

If a particular x-value makes both numerator & denominator 0 that factor will cancel and we will have a <u>hole</u> in the graph

If an x-value makes only the denominator equal to 0, the graph will have a vertical asymptote @ that x-value

End behavior (what happens to $f(x)$ as $x \to \pm\infty$)

Look @ ratio of lead terms.

- If denominator has a higher degree, horizontal asymptote $y=0$.
  ($f(x) \to 0$ as $x \to \pm\infty$)

  E.g. $f(x) = \dfrac{-3x^5 - 2x^2 + 3}{5x^9 + 2x}$

  $\dfrac{-3x^5}{5x^9} = \dfrac{-3}{5x^4} \to 0$ as $x \to \pm\infty$

- If numerato & denominator have the same degree, then horizontal asymptote is $y =$ ratio of leading coefficients

  e.g. $f(x) = \dfrac{5x^2 - 4}{10x^2 + 3x}$ H.A. is $y = \frac{5}{10}$

  $\dfrac{5x^2}{10x^2} = \dfrac{5}{10}$ $f(x) \to \frac{5}{10}$ as $x \to \pm\infty$

- If numerator has a larger degree, perform long division.

If deg is 1 higher $\rightarrow$ linear "oblique" asymptote

If deg is 2 higher $\rightarrow$ parabolic asymptote

3 $\rightarrow$ cubic asymptote

. . .

$$f(x) = \frac{x^2-7x+6}{x+5} = \frac{(x-6)(x-1)}{x+5}$$

num > denom

$$\begin{array}{r|l} & x-12 \\ x+5 & x^2-7x+6 \\ & -(x^2+5x) \\ & \quad -12x+6 \\ & \quad -(-12x-60) \\ & \qquad 66 \end{array}$$

quotient is $\boxed{x-12} + \frac{66}{x+5}$

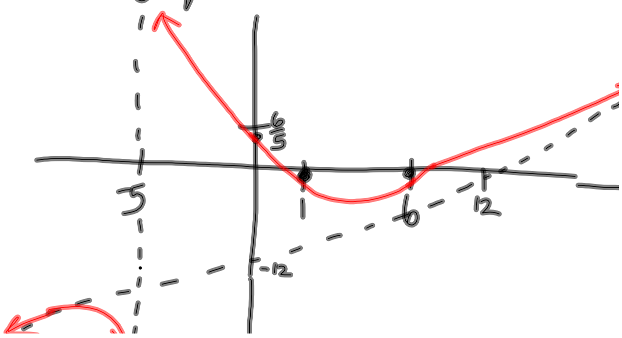
$\frac{66}{x+5} \rightarrow 0$ as $x \to \pm\infty$

zeros: 1, 6

y-int: $(0, \frac{6}{5})$

oblique asymptote: $y = x-12$

vertical asymptote: $x = -5$



$$f(x)=\frac{2x(x+3)(x-1)}{(x-4)(x+1)(x+6)}$$

$f(-2)=\frac{-+-}{--+}$

$f(2)=+++ > 0$

$-++ < 0$

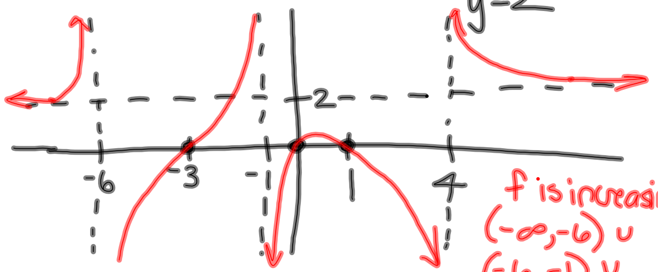
zeros: 0, −3, 1

y-int: (0,0)

vertical asymptotes: $x=4, x=-1, x=-6$

ratio of lead terms:

$\frac{2x\cdot x\cdot x}{x\cdot x\cdot x}=\frac{2x^3}{x^3}=2 \Rightarrow$ horizontal asymptote $y=2$



f is increasing: $(-\infty,-6) \cup (-6,-1) \cup (-1, x)$

f is decreasing $(x, 4) \cup (4, \infty)$

as $x \to -\infty$, $f(x) \to 2$

$x \to \infty$, $f(x) \to 2$

as $x \to -6^-$, $f(x) \to \infty$; as $x \to -6^+$, $f(x) \to -\infty$

from the left — from the right

as $x \to -1^-$, $f(x) \to \infty$; as $x \to -1^+$, $f(x) \to -\infty$

$x \to 4^-$, $f(x) \to -\infty$; as $x \to 4^+$, $f(x) \to \infty$

HW:

3.5 # 7-25 odd