

1. Find the polynomial of least degree that has 6,  $4i$ , and  $-\sqrt{5}$  as three of its zeros. Give the polynomial in factored form. Do NOT multiply it out.

$$f(x) = (x-6)(x-4i)(x+4i)(x+\sqrt{5})(x-\sqrt{5})$$

2. List the possible rational zeros of the function  $f(x) = x^4 + 5x^3 + 5x^2 - 5x - 6$

$$\pm 6, \pm 3, \pm 2, \pm 1 \quad | \quad \text{ea}$$

3. Find the zeros of the function  $f(x) = x^4 + 5x^3 + 5x^2 - 5x - 6$ , and write  $f(x)$  as a product of linear factors. Hint:  $-2$  is one of the zeros.

$$\begin{array}{r|rrrrr} -2 & 1 & 5 & 5 & -5 & -6 \\ & & -2 & -6 & 2 & 6 \\ \hline & 1 & 3 & -1 & -3 & 0 \\ & x^3 & x^2 & x & & \end{array}$$

$$\begin{aligned} &(x+2)(x^3+3x^2-x-3) \\ &\quad x^2(x+3)-1(x+3) \\ &\quad (x+3)(x^2-1) \end{aligned}$$

$$f(x) = (x+2)(x+3)(x-1)(x+1)$$

### 3.5 Rational Functions

$$f(x) = \frac{p(x)}{q(x)}$$

**x-intercepts** are the points  $(x,0)$  at which the graph crosses the x-axis.

To find: set numerator = 0 and solve for x.

**vertical asymptotes** are vertical lines which the graph of a function approaches but never touches.

They occur at x-values for which the function is undefined. Always have equation of the form  $x=c$ .

To find: set the denominator = 0 and solve for x.\*

\*except when the same values make both the numerator and the denominator = 0, in which case there is not an asymptote, but instead a hole in the graph at that x-value.

**y-intercepts** are the points  $(0,y)$  at which the graph crosses the y-axis. The graph of a function should only have one of these if any.

To find: plug 0 in for x.

**horizontal asymptotes** are horizontal lines ( $y=c$ ) that determine the end behavior of the graph a function. As  $x$  goes to  $+\infty$  (the far right and left sides of your graph), the graph will approach these lines, may cross them, and may even lie exactly on them.

To find: look at the ratio of lead terms. If the degree of the numerator and denominator is the same, the H.A. is the ratio of leading coefficients. If the degree of the numerator is smaller than the degree of the denominator, the H.A. is  $y=0$ . If there are radicals or fractions present, take those into account following order of operations.

**oblique/slant asymptotes** are diagonal lines ( $y=mx+b$ ) that determine the end behavior of the graph of a function. A graph will have either horizontal or oblique asymptotes, but never both.

To find: these occur when the degree of the numerator is 1 higher than the degree of the denominator.

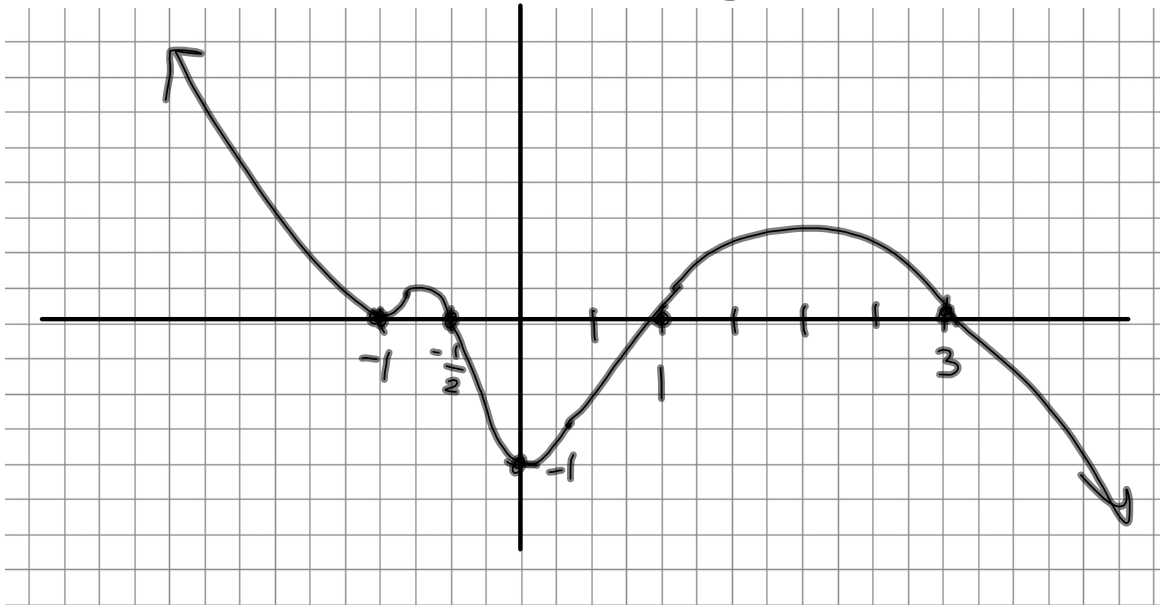
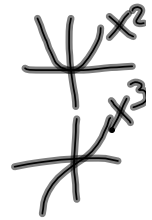
Found using long division (numerator divided by denominator)

$$f(x) = -\frac{2}{3}(x+1)^2(x-3)(x+\frac{1}{2})(x-1)^3$$

zeros/  
(mult)      -1                      3                      -1/2                      1  
                  (2)                      (1)                      (1)                      (3)

y-int:  $-\frac{2}{3}(1)^2(-3)(\frac{1}{2})(-1)^3 = -\frac{2}{3} \cdot \frac{-3}{2}(-1) = -1$  (0, -1)

Lead term:  $-\frac{2}{3}(x)^2 \cdot x \cdot x \cdot x^3 = -\frac{2}{3}x^7$



Zeros: -4, 3

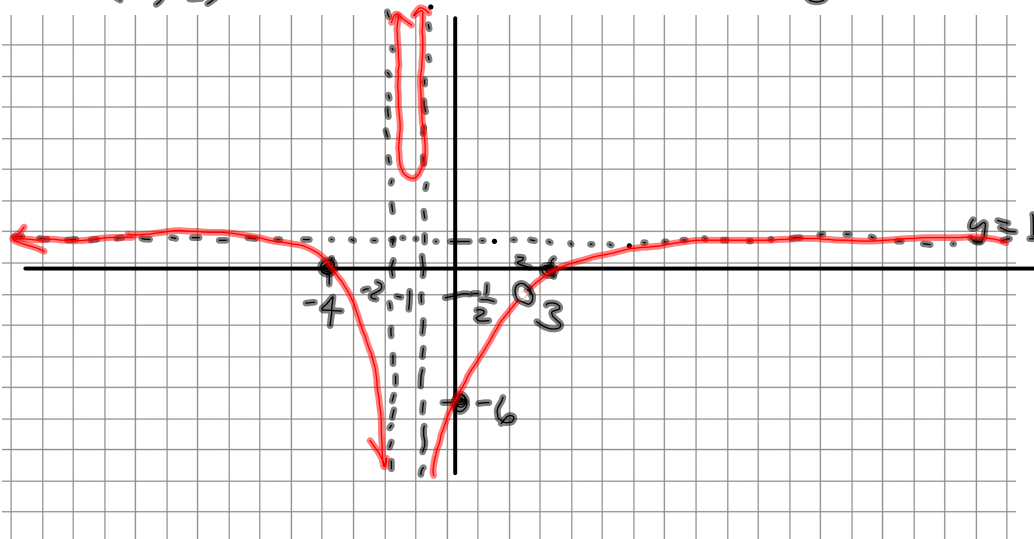
vert. asymptotes:  $x = -1, x = -2$

y-intercept: (0, -6)

end behavior:  $\frac{x^2}{x^2} = 1$       horiz. asymptote  $y = 1$

$f(x) = \frac{(x-2)(x+4)(x-3)}{(x-2)(x+1)(x+2)}$

\*hole @  $(2, \frac{(2+4)(2-3)}{(2+1)(2+2)}) = (2, -\frac{1}{2})$



$$f(x) = \frac{(x-4)(x+3)}{x-2}$$

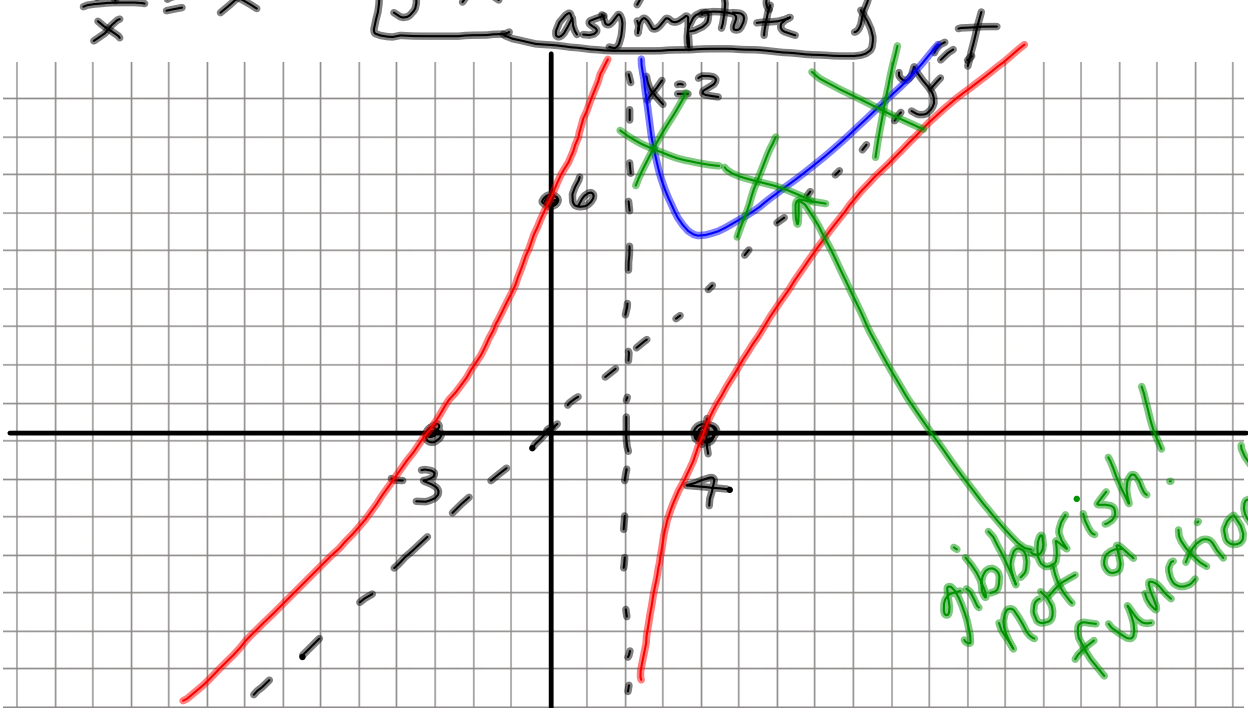
x-intercepts:  $(4,0), (-3,0)$   
 y-int:  $(0,6)$

V.A.:  $x=2$

end behavior:

$$\frac{x^2}{x} = x$$

$y=x$  slant/oblique asymptote



$$y = \frac{(2x-1)(x-4)}{(x-2)(x+1)}$$

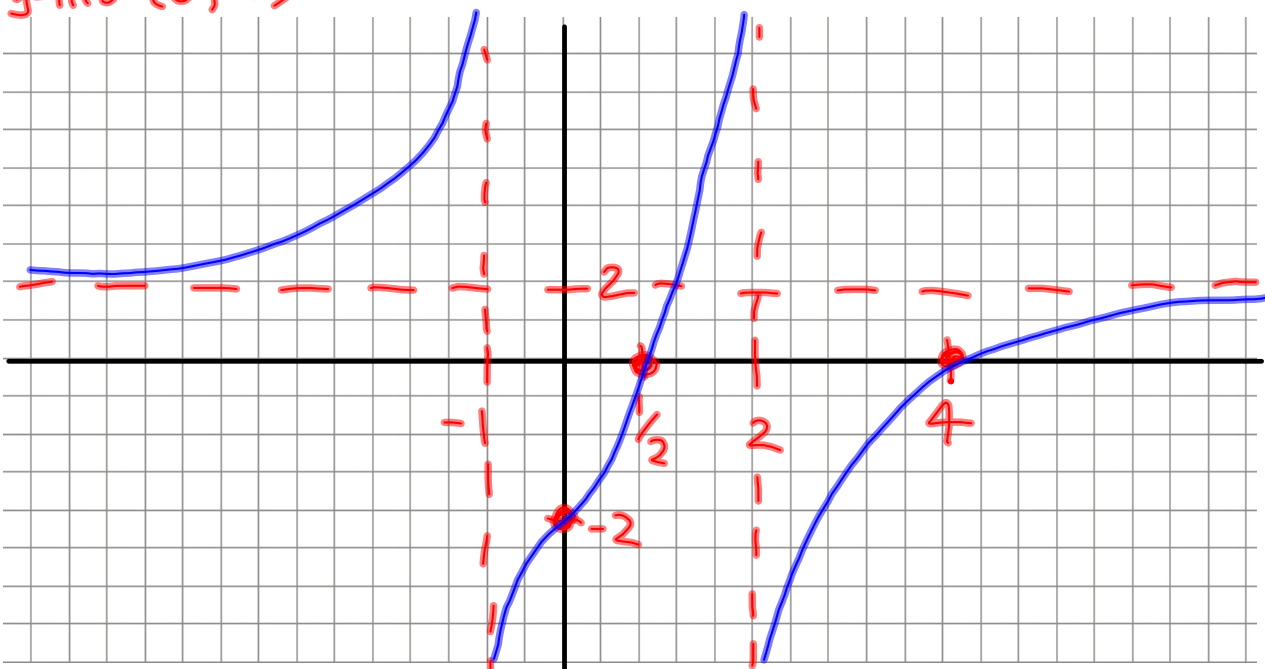
V.A.:  $x=2, x=-1$

end behavior:

$$\frac{2x^2}{x^2} = 2$$

H.A.:  $y=2$

zeros:  $\frac{1}{2}, 4$   
 y-int:  $(0,-2)$



**Homework:**

**3.5 #27-67 odd (already assigned)**

(read 3.6 on polynomial &  
rational inequalities)