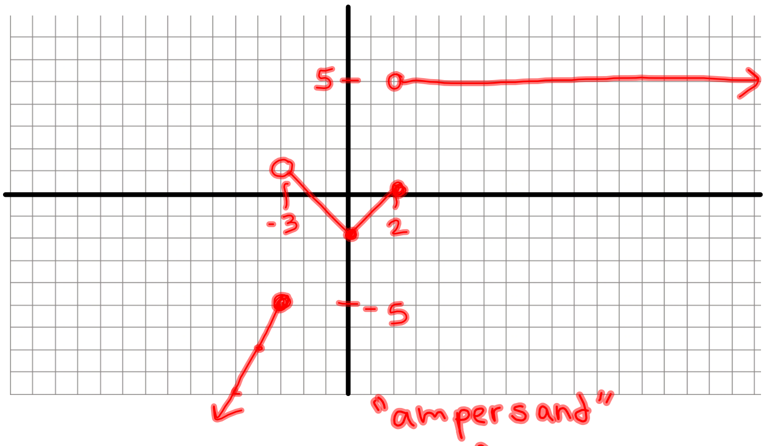
Review:

Graph the piecewise function:

$$f(x)=\begin{cases} 2x+1, & x\le -3 \\ |x|-2, & -3<x\le 2 \\ 5, & x>2 \end{cases}$$



"ampersand"

&

Given the polynomial f(x)= $2x^3+x^2+18x+9$

What does Descartes' Rule of Signs tell us about the number of positive real zeros & negative real zeros?

positive? none (no sign changes)

$f(-x)=-2x^3+x^2-18x+9$

$\boxed{\text{3 or 1 negative real zeros}}$

3.6 Polynomial and Rational Inequalities

Linear: $2x+1>5$

$2x>4$

$\{x \mid x>2\}$

$(2,\infty)$

abs. value.

$|2x+1|>5$

$2x+1>5$ or $2x+1<-5$

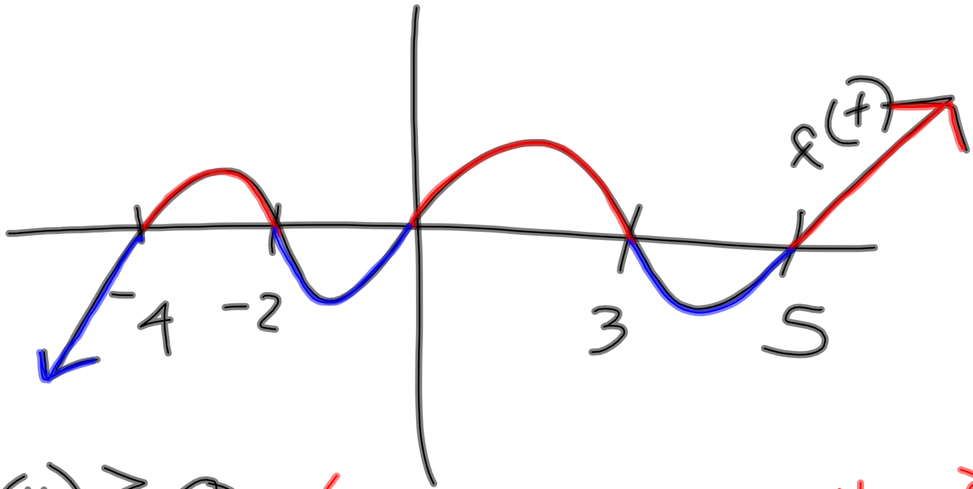
$2x>4$ $\quad$ $2x<-6$

$x>2$ $\quad$ $x<-3$

$\{x \mid x<-3 \text{ or } x>2\}$

-3 $\quad$ 2

$(-\infty,-3)\cup(2,\infty)$



$f(x) \geq 0$ (where is f(x) positive?)

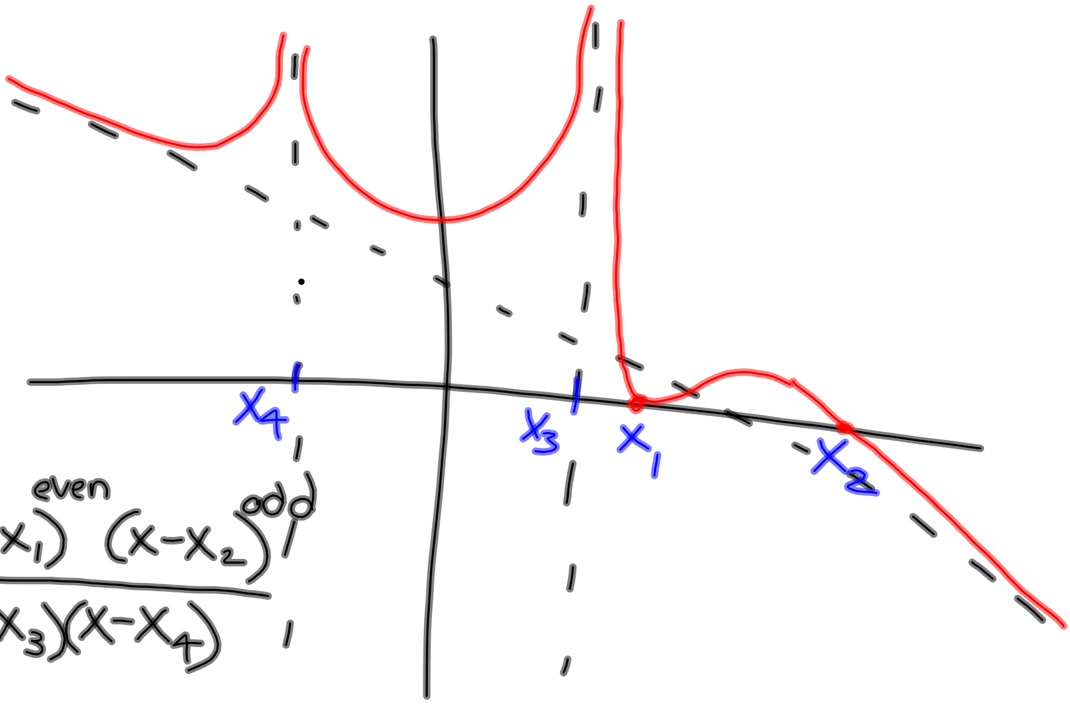
$[-4,-2] \cup [0,3] \cup [5,\infty)$

$f(x) < 0$ $(-\infty,-4) \cup (-2,0) \cup (3,5)$

$$\frac{5x^3+7x^2+3x-1}{2x+4} \geq \frac{3x^2+7}{2x^3-5x}$$

hard to solve algebraically

easier to compare to 0
(where is it positive or negative?)



$$\frac{(x-x_1)^{\text{even}}(x-x_2)^{\text{odd}}}{(x-x_3)(x-x_4)}$$

The only x-values @ which the value of f(x) <u>can</u> change from positive to negative (or negative to positive) are @ zeros & vertical asymptotes (but it doesn't have to change)

$$\underbrace{(x+4)(x-3)(x+1)}_{\text{zeros: } -4,\,3,\,-1} < 0$$

f(-5) f(-2) f(0) f(4)

−4 −1 3

− + − +

$$\boxed{(-\infty,-4)\cup(-1,3)}$$

$\{x \mid x<-4 \text{ or } -1<x<3\}$

$$x^2+6x \geq 7$$

$0 \leq f(x)$

$\boxed{f(x) \geq 0}$

1. rearrange to compare to zero.

$x^2+6x-7 \geq 0$

2. factor to find zeros (or vertical asymptotes)

$(x+7)(x-1) \geq 0$

zeros: $-7, 1$

3. split real number line into intervals according to #'s found in step 2; test a value in each interval for +/−

f(-8) f(0) f(2)

−7 1

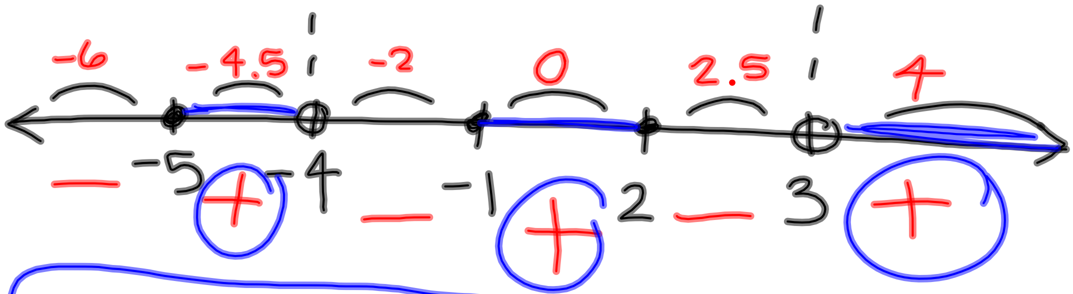
+ − +

$$\boxed{(-\infty,-7]\cup[1,\infty)}$$

$$\begin{array}{ccc} -5 & -1 & 2 \end{array}$$

$$\frac{(x+5)(x+1)(x-2)}{(x-3)(x+4)} \geq 0$$

$$\begin{array}{cc} 3 & -4 \end{array}$$

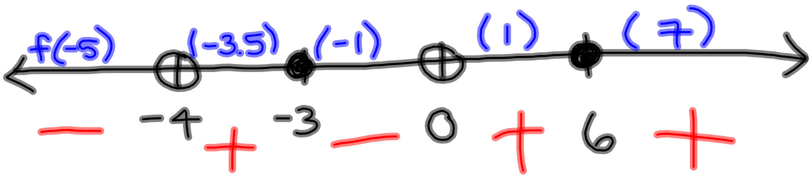


$$\boxed{[-5,-4) \cup [-1,2] \cup (3,\infty)}$$

$$\begin{array}{cc} 6 & -3 \end{array}$$

$$\frac{(x-6)^2(x+3)}{x(x+4)} \leq 0$$

$$\begin{array}{cc} 0 & -4 \end{array}$$



$$\boxed{(-\infty,-4) \cup [-3,0)}$$

3.6

56. $\frac{3}{x^2-4} \le \frac{5}{x^2+7x+10}$

$\frac{3}{x^2-4}\cdot\frac{x+5}{x+5} - \frac{5}{x^2+7x+10}\cdot\frac{x-2}{x-2} \le 0$

$(x-2)(x+2) \quad (x+2)(x+5)$

$\frac{3(x+5)-5(x-2)}{(x-2)(x+2)(x+5)} \le 0$

$\frac{3x+15-5x+10}{(x-2)(x+2)(x+5)} \le 0$

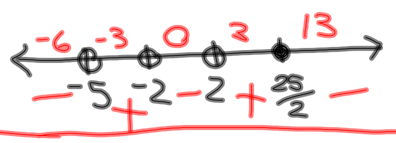
$\frac{-2x+25}{(x-2)(x+2)(x+5)} \le 0$

$2 \quad -2 \quad -5$

$-2x+25=0$

$-2x=-25$

$x=\frac{25}{2}$



$\boxed{(-\infty,-5)\cup(-2,2)\cup\left[\frac{25}{2},\infty\right)}$

3.6

31-39 odd

47, 53-61 odd