Precalculus Quiz #6 Name:_____
Brewer 12 April 2013

Given the rational function
$f(x) = \frac{x(x-4)(x+2)}{(x-2)(x+5)(x-6)}$ $= \frac{x^3 \ldots}{x^3 \ldots}$

1. Determine the zeros of the function.

$0, 4, -2$

2. Determine the y-intercept of the function.

$(0,0)$

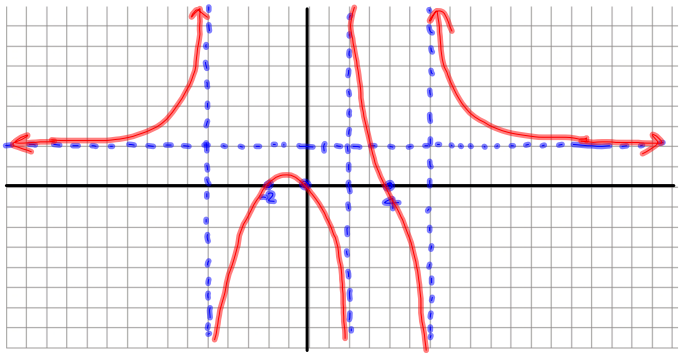
3. Determine the equations of any vertical asymptotes of the function.

$x=2, x=-5, x=6$

4. Determine the equations of any horizontal or oblique asymptotes of the function.

$\frac{x^3}{x^3} = 1 \qquad y=1$

5. Graph the function.



1. Given the polynomial $f(x) = 2x^3 + x^2 + 18x + 9$,

a. (5 points) What does **Descartes' Rule of Signs** tell us about the number of positive real zeros and number of negative real zeros?

positive: $f(x)$ has no sign change $\Rightarrow$ 0 positive real zeros

negative: $f(-x) = 2(-x)^3 + (-x)^2 + 18(-x) + 9$
$= -2x^3 + x^2 - 18x + 9$

3 sign Δ's $\Rightarrow$ 3 or 1 negative real zeros

b. (3 points) List all **possible** rational zeros of the polynomial.

$\frac{\text{factors of constant term}}{\text{factors of leading coeff.}} = \frac{\pm 9, \pm 3, \pm 1}{\pm 2, \pm 1} = \pm\frac{9}{2}, \pm 9, \pm\frac{3}{2}, \pm 3, \pm\frac{1}{2}, \pm 1$

c. (5 points) Given that $-\frac{1}{2}$ is a zero of the polynomial, use synthetic division to **find all other zeros**.

$-\frac{1}{2}$ | 2 | 1 | 18 | 9
| | | -1 | 0 | -9
| 2 | 0 | 18 | 0

$f(x) = (x+\frac{1}{2})(2x^2+18)$

$2x^2 + 18 = 0$
$2x^2 = -18$
$x^2 = -9$
$x = \pm 3i$

d. (2 points) Write the polynomial as a product of linear factors.

$f(x) = (x+\frac{1}{2})(x+3i)(x-3i)$

2. Given the polynomial $(x) = -\frac{1}{3}\left(x+\frac{3}{2}\right)^2 (x-1)^3(x-2)^4$,

a. (5 points) Find the real **zeros** and state the **multiplicity** of each.

zeros: $-3/2$ 1 2

mult: 2 3 4

even odd even

b. (5 points) State the **lead term** and make a sketch depicting the end behavior of the graph.

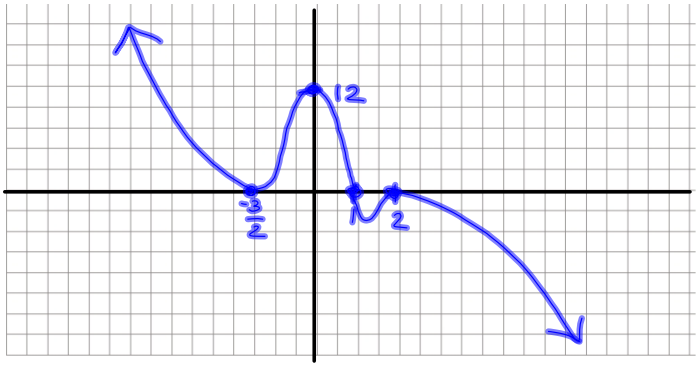
$-\frac{1}{3}x^2 \cdot x^3 \cdot x^4 = \boxed{-\frac{1}{3}x^9}$

c. (3 points) State the y-intercept as an **ordered pair**.

$-\frac{1}{3}\left(\frac{3}{2}\right)^2(-1)^3(-2)^4 = +\frac{1}{3}\cdot\frac{3^2}{2^2}\cdot+1\cdot\frac{2^4}{1} = 3\cdot 2^2 = 12$

$(0,12)$

d. (6 points) Graph the polynomial. **Label** x- and y-intercepts.



3. (8 points) **Find the equation of a polynomial** of lowest degree with 4, $\sqrt{2}$, and $3-2i$ as three of its zeros. Leave your answer in factored form.

$-\sqrt{2}$ $3+2i$

$(x-4)(x-\sqrt{2})(x+\sqrt{2})(x-(3-2i))(x-(3+2i)) = f(x)$

4.(8 points) Use the **intermediate value theorem** to determine if the function $f(x) = 4x^3 - 3x + 3$ has a zero between -2 and -1.

If $f(a)>0$ & $f(b)<0$ (or vice versa),
there is some $c \in (a,b)$ such that $f(c) = 0$.

$f(-2) = 4(-8) - 3(-2) + 3 = -32 + 6 + 3 < 0$

$f(-1) = 4(-1) - 3(-1) + 3 = -4 + 3 + 3 > 0$

$\Rightarrow$ IVT guarantees a zero between -2 & -1.

5.(8 points) Solve the polynomial inequality. Give your answer in interval notation.

$4x^2 > 3x + 7$

$4x^2 - 3x - 7 > 0$

$4x^2 + 4x - 7x - 7 > 0$

$4x(x+1) - 7(x+1) > 0$

$(x+1)(4x-7) > 0$

zeros: -1 & $7/4$

$(-\infty, -1) \cup \left(\frac{7}{4}, \infty\right)$

$f(-2)$ $f(0)$ $f(2)$

$+$ -1 $-$ $7/4$ $+$

6. Given the rational function $f(x) = \frac{x^3 + 6x^2 - 4x - 24}{x^2 + 2x - 3}$

$x^2(x+6) - 4(x+6)$

a. (3 points) **Factor** the function completely.

$\frac{(x^2-4)(x+6)}{(x+3)(x-1)} = \frac{(x-2)(x+2)(x+6)}{(x+3)(x-1)}$

b. (5 points) State the real **zeros** and the **multiplicity** of each.

$-2, 2, -6$ – all mult 1

c. (2 points) State the y-intercept as an **ordered pair**.

$\frac{-24}{-3} = 8 \quad (0, 8)$

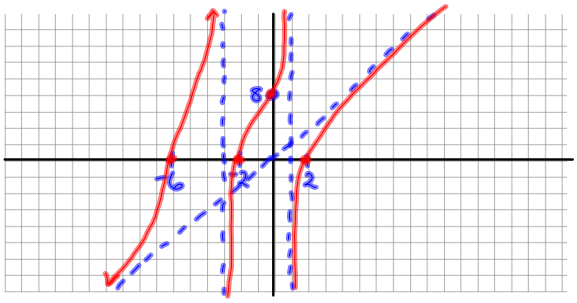
d. (4 points) Find the **equation** of any vertical asymptotes of the function.

$x = -3 \; ; \; x = 1$

e. (5 points) Find the **equation** of any horizontal or oblique asymptotes.

$\frac{x^3}{x^2} = x \qquad y = x$

f. (5 points) Graph the rational function. **Label** all x- and y-intercepts and any asymptotes.



7. (8 points) Solve the rational inequality. Give your answer in interval notation.

$\frac{x-4}{x+2} \geq 3$

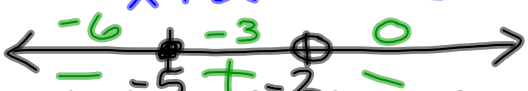
$\frac{x-4}{x+2} - 3 \geq 0 \quad \cdot \frac{x+2}{x+2}$

$\frac{x-4-3(x+2)}{x+2} \geq 0$

$\frac{x-4-3x-6}{x+2} \geq 0$

$\frac{-2x-10}{x+2} \geq 0$

$\frac{-2(x+5)}{x+2} \geq 0$



$[-5, -2)$

8. (10 points) Find an equation of variation where y varies jointly as the square of x and the square of z and inversely as w, and $y = 50$ when $x = 2$, $z = 5$, and $w = 10$.

$y = \frac{k x^2 z^2}{w}$

$50 = \frac{k(2)^2(5)^2}{10}$

$k = 50 \cdot \frac{10}{2^2 \cdot 5^2} = 5$

$y = \frac{5x^2z^2}{w}$