

4.1 Inverse Functions

Recall:

f is a function if each input value (x) has exactly one output $f(x)$

Functions pass the vertical line test.

f is a one-to-one function if, in addition, each y corresponds to only one x .

One-to-one functions pass both the horizontal line test and the vertical line test.

Formally, a function is one-to-one if different inputs have the same output, i.e.

if $a \neq b$, then $f(a) \neq f(b)$,

Or equivalently, f is one-to-one if when the outputs are the same, the inputs are the same, i.e.

if $f(a) = f(b)$, then $a = b$.

Proving that a function is one-to-one v. proving that a function is not one-to-one
(problems 17-24 from section 4.1)

Given $f(x) = x^2 - 5$

To show that $f(x)$ is not one-to-one, it is enough to provide a single counter-example, i.e. 2 different inputs that yield the same output

$$f(2) = 2^2 - 5 = -1$$

$$f(-2) = (-2)^2 - 5 = -1$$

$$f(x) = -2x^3 + 1$$

To show that $f(x)$ is one-to-one, we must prove it in general.

$$f(a) = f(b)$$

$$-2a^3 + 1 = -2b^3 + 1$$

$$-2a^3 = -2b^3$$

$$a^3 = b^3$$

$$a = b$$

Start w/
 $f(a) = f(b)$ &
 manipulate algebraically
 until you get $a = b$

If a function is one-to-one, then it has an inverse.

Interchanging the first and second coordinates of each ordered pair in a relation produces the inverse function.

$$f(x) = \{(1, 2), (3, 4), (5, 6), (7, 8)\}$$

$$f^{-1}(x) = \{(2, 1), (4, 3), (6, 5), (8, 7)\}$$

If a relation is defined by an equation, interchanging the variables produces an equation of the inverse relation.

$$y = -2x^3 + 1$$

$$x = -2y^3 + 1$$

The domain of a one-to-one function f is the range of the inverse f^{-1} .

The range of a one-to-one function f is the domain of the inverse f^{-1} .

$$f^{-1}(x) \neq \frac{1}{f(x)}$$

Obtaining the formula for an inverse:

1. Replace $f(x)$ with y
2. Interchange x and y
3. Solve for y
4. Replace y with $f^{-1}(x)$

$$f(x) = -2x^3 + 1$$

$$y = -2x^3 + 1$$

$$x = -2y^3 + 1$$

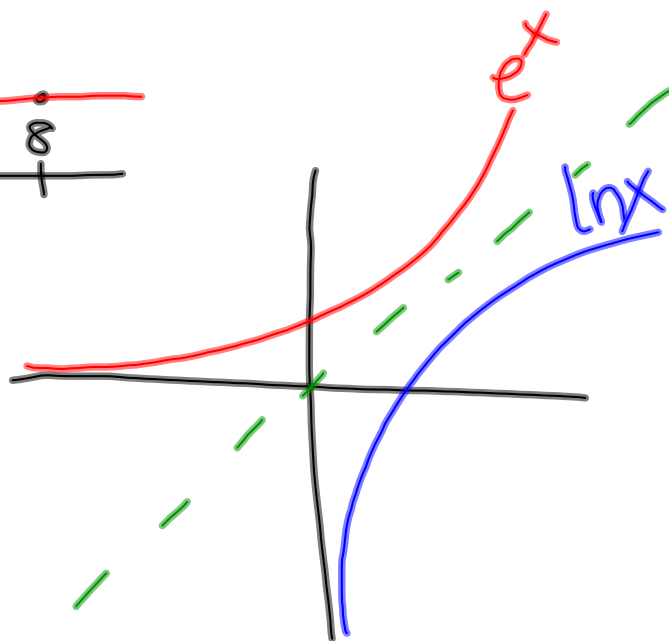
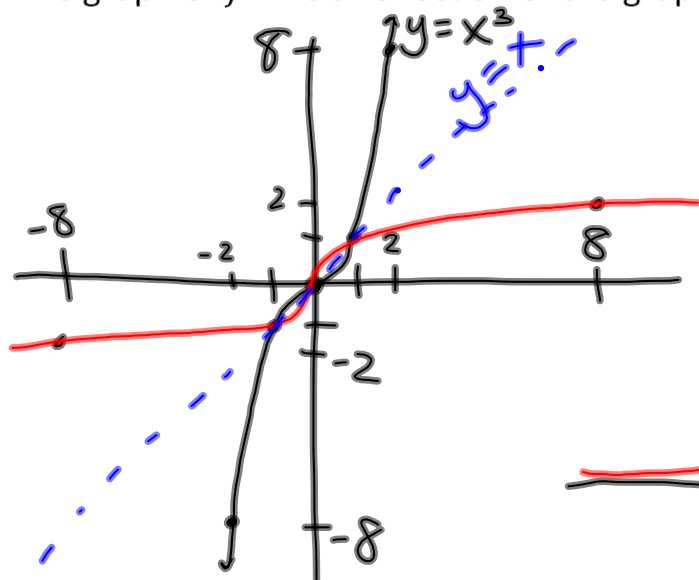
$$2y^3 = 1 - x$$

$$y^3 = \frac{1 - x}{2}$$

$$y = \sqrt[3]{\frac{1 - x}{2}}$$

$$f^{-1}(x) = \sqrt[3]{\frac{1 - x}{2}}$$

The graph of f^{-1} is a reflection of the graph of f across the line $y = x$



If f & g are inverses, then

$$(f \circ g)(x) = x \quad \text{for all } x \text{ in the domain of } g$$

AND

$$(g \circ f)(x) = x \quad \text{for all } x \text{ in the domain of } f$$

86. $f(x) = \sqrt[3]{x+4}$; $f^{-1}(x) = x^3 - 4$

use composition to show f^{-1} is as given.

$$(f \circ f^{-1})(x) = \sqrt[3]{(x^3 - 4) + 4} = \sqrt[3]{x^3} = x$$

$$(f^{-1} \circ f)(x) = (\sqrt[3]{x+4})^3 - 4 = x + 4 - 4 = x$$

$$88. f(x) = \frac{x+6}{3x-4}, f^{-1}(x) = \frac{4x+6}{3x-1}$$

$$(f \circ f^{-1})(x) = \frac{\frac{4x+6}{3x-1} + \cancel{6} \cdot \frac{(3x-1)}{3x-1}}{\frac{3(\frac{4x+6}{3x-1}) - \cancel{4} \cdot \frac{(3x-1)}{3x-1}}$$

$$= \frac{(4x+6 + 6(3x-1))}{3x-1}$$

$$\frac{(3(4x+6) - 4(3x-1))}{3x-1}$$

$$= \frac{4x+6 + 18x - 6}{\cancel{3x-1}} \cdot \frac{\cancel{3x-1}}{12x+18-12x+4}$$

$$= \frac{\cancel{22}x}{\cancel{22}} = x$$

$$(f^{-1} \circ f)(x) = \dots = x$$

4.1 HW

17-23 odd

59-63 odd

77-85 odd