4.1 Inverse Functions

Recall:

f is a function if each input value (x) has exactly one output $f(x)$

Functions pass the vertical line test.

f is a one-to-one function if, in addition, each y corresponds to only one x.

One-to-one functions pass both the horizontal line test and the vertical line test.

Formally, a function is one-to-one if different inputs have the same output, i.e.

$if\ a \neq b, then\ f(a) \neq f(b),$

Or equivalently, f is one-to-one if when the outputs are the same, the inputs are the same, i.e.

$if\ f(a) = f(b), then\ a = b.$

Proving that a funcon *is* one-to-one v. proving that a funcon *is not* one-to-one
(problems 17-24 from secon 4.1)

Given $f(x) = x^2 - 5$

To prove that f is not one-to-one, we need to find a counter-example, i.e. 2 different inputs that yield the same output.

$f(1) = 1^2 - 5 = -4$

$f(-1) = (-1)^2 - 5 = -4$

Given $f(x) = -2x^3 + 1$.

$$f(a) = f(b)$$
$$-2a^3 + 1 = -2b^3 + 1$$
$$-2a^3 = -2b^3$$
$$a^3 = b^3$$
$$a = b$$

To prove that f is one-to-one, start w/ $f(a) = f(b)$ & manipulate algebraically until you get $a = b$.

If a funcon is one-to-one, then it has an inverse.

Interchanging the first and second coordinates of each ordered pair in a relaon produces the inverse funcon.

$$f(x) = \{(1,2), (3,4), (5,6), (7,8)\}$$
$$f^{-1}(x) = \{(2,1), (4,3), (6,5), (8,7)\}$$

If a relaon is defined by an equaon, interchanging the variables produces an equaon of the inverse relaon.

$$y = -2x^3 + 1$$

inverse: $x = -2y^3 + 1$

The domain of a one-to-one function f is the range of the inverse f^{-1}.

The range of a one-to-one function f is the domain of the inverse f^{-1}.

$$f^{-1}(x) \neq \frac{1}{f(x)}$$

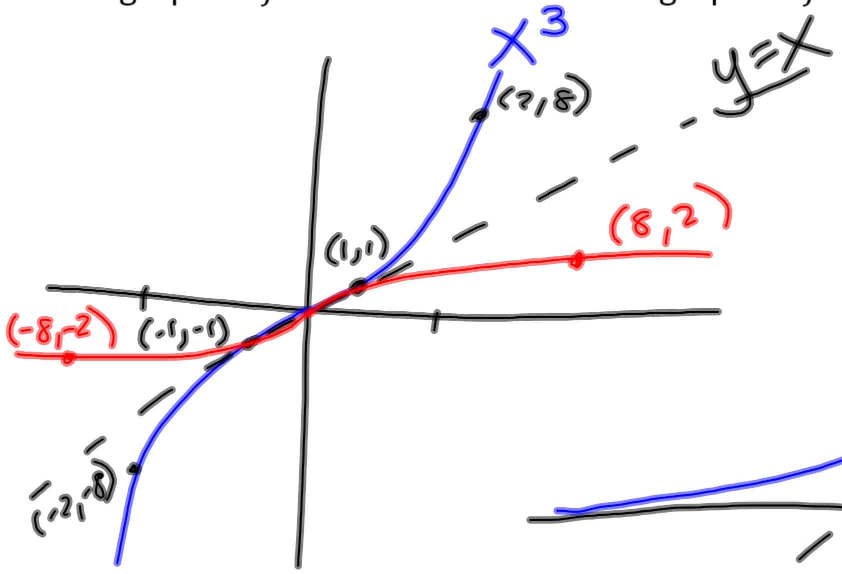
Obtaining the formula for an inverse:

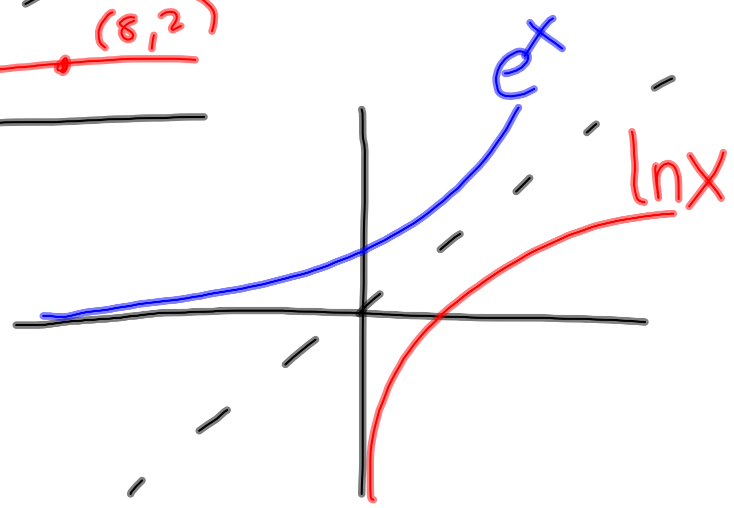
1. Replace $f(x)$ with y
2. Interchange x and y
3. Solve for y
4. Replace y with $f^{-1}(x)$

$$f(x) = -2x^3 + 1$$
$$y = -2x^3 + 1$$
$$x = -2y^3 + 1$$
$$2y^3 = 1 - x$$
$$y^3 = \frac{1-x}{2}$$
$$y = \sqrt[3]{\frac{1-x}{2}}$$

$$f^{-1}(x) = \sqrt[3]{\frac{1-x}{2}}$$

The graph of f^{-1} is a reflection of the graph of f across the line $y = x$





If f & g are inverses, then

$(f \circ g)(x) = x$ for all x in the domain of g

AND

$(g \circ f)(x) = x$ for all x in the domain of f

86. $f(x) = \sqrt[3]{x+4}$; $f^{-1}(x) = x^3 - 4$

use composition to show f^{-1} is as given.

$$(f \circ f^{-1})(x) = \sqrt[3]{(x^3-4)+4} = \sqrt[3]{x^3} = x$$

$$(f^{-1} \circ f)(x) = \left(\sqrt[3]{x+4}\right)^3 - 4 = x+4-4 = x$$

88. $f(x)=\frac{x+6}{3x-4}$, $f^{-1}(x)=\frac{4x+6}{3x-1}$

$$(f\circ f^{-1})(x)=\frac{\frac{4x+6}{3x-1}+6\frac{(3x-1)}{3x-1}}{3\frac{(4x+6)}{3x-1}-4\frac{(3x-1)}{3x-1}}$$

$$=\frac{\left(\frac{4x+6+6(3x-1)}{3x-1}\right)}{\left(\frac{3(4x+6)-4(3x-1)}{3x-1}\right)}$$

$$=\frac{4x+6+18x-6}{\cancel{3x-1}}\cdot\frac{\cancel{3x-1}}{12x+18-12x+4}$$

$$=\frac{\cancel{22}x}{\cancel{22}}=x$$

$$(f^{-1}\circ f)(x)=\ \cdots\ =x$$

4.1 HW

17-23 odd

59-63 odd

77-85 odd