4.2 Exponential Functions

$f(x) = a^x$ $a > 0$ excludes $(-1)^{1/2}$

base exponent $a \neq 1$ excludes $1^x = 1$ constant

Note: the variable is in the exponent, unlike power functions/polynomials

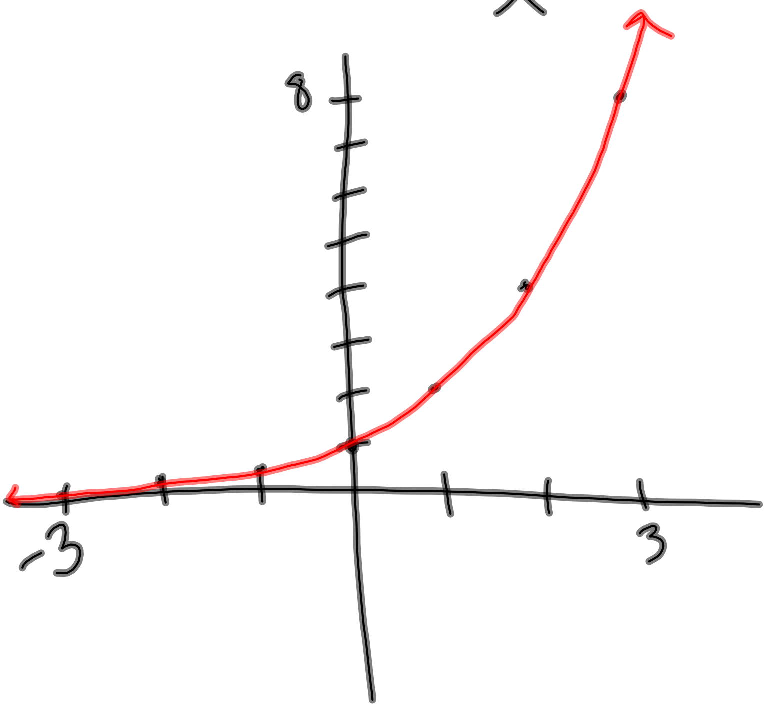
$f(x) = x^2$ power/poly

$f(x) = 2^x$ exp.

$f(x) = 2^x$

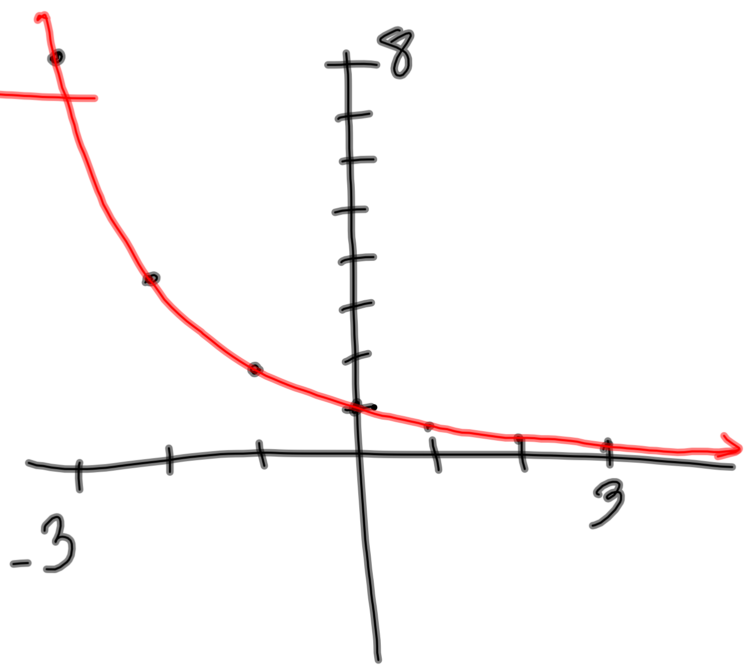
$x^{-n} = \frac{1}{x^n}$

x	2^x
-3	$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$
-2	$2^{-2} = \frac{1}{2^2} = \frac{1}{4}$
-1	$2^{-1} = \frac{1}{2^1} = \frac{1}{2}$
0	$2^0 = 1$
1	$2^1 = 2$
2	$2^2 = 4$
3	$2^3 = 8$



$$f(x) = \left(\frac{1}{2}\right)^x = \frac{1}{2^x} = 2^{-x}$$

x	$\left(\frac{1}{2}\right)^x = \frac{1}{2^x}$
-3	$\frac{1}{2^{-3}} = 2^3 = 8$
-2	$\frac{1}{2^{-2}} = 2^2 = 4$
-1	$\frac{1}{2^{-1}} = 2^1 = 2$
0	$\frac{1}{2^0} = \frac{1}{1} = 1$
1	$\frac{1}{2^1} = \frac{1}{2}$
2	$\frac{1}{2^2} = \frac{1}{4}$
3	$\frac{1}{2^3} = \frac{1}{8}$

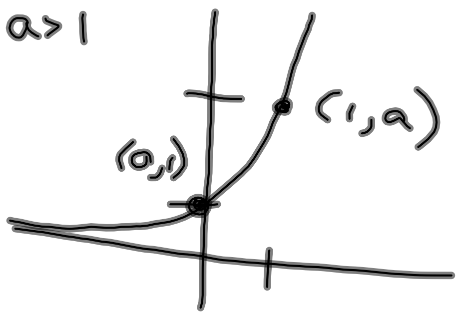


Properties of Exponential Functions

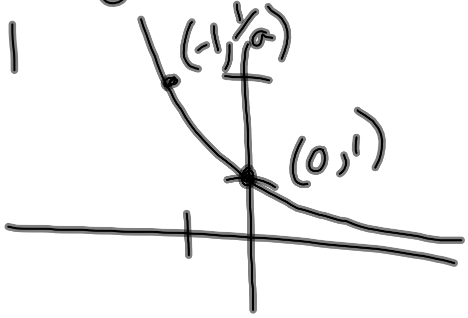
$$f(x) = a^x \quad , \quad a > 0 \, , \, a \neq 1$$

- continuous
- one-to-one
- Domain: $(-\infty, \infty)$
- Range: $(0, \infty)$
- Horizontal asymptote: $y = 0$
- y-intercept: $(0,1)$
- increasing if $a > 1$
- decreasing if $0 < a < 1$

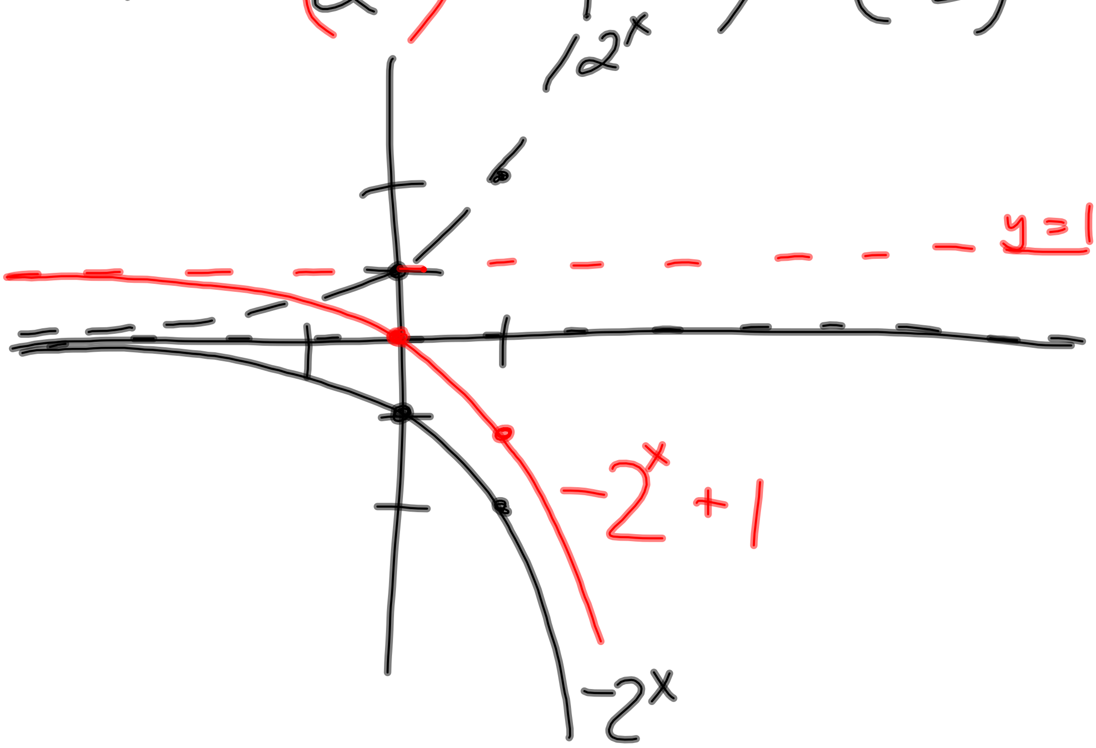
$a > 1$



$0 < a < 1$



$$f(x) = -(2^x) + 1 \neq (-2)^x + 1$$



Application: Compound Interest

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

A = amount of money
P = principal (initial investment)
t = time in # of years
r = interest rate (decimal)
n = # of times interest is compounded per year

Example:
\$100 @ 5% interest compounded quarterly for 1 year

$$A(t) = P\left(1+\frac{r}{n}\right)^{nt}$$
$$= 100\left(1+\frac{0.05}{4}\right)^{4\cdot 1}$$
$$\approx \$105.09$$

If we invest \$1 at 100% interest for 1 year,

$$A=P\left(1+\frac{r}{n}\right)^{nt} = 1\left(1+\frac{1}{n}\right)^{n\cdot 1}$$

$$A=\left(1+\frac{1}{n}\right)^{n}$$

n	$A(n)=\left(1+\frac{1}{n}\right)^n$
1 annually	$\left(1+\frac{1}{1}\right)^1 = \$2$
2 semiannually	$\left(1+\frac{1}{2}\right)^2 = 2.25$
4 quarterly	$\left(1+\frac{1}{4}\right)^4 = 2.4414$
12 montly	$\left(1+\frac{1}{12}\right)^{12} = 2.6130$
365 daily	$\left(1+\frac{1}{365}\right)^{365} = 2.7146$
8760 hourly	$\left(1+\frac{1}{8760}\right)^{8760} = 2.7181$
$\to \infty$	$e \approx 2.7182818284\ldots$

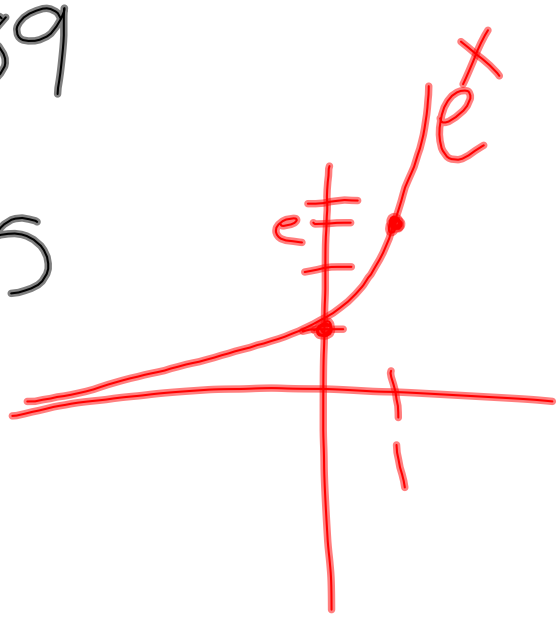
e is an irrational # like π

$e = 2.718$

$e^2 = e\hat{\ }2 \approx 7.389$

$e^{-0.23} \approx 0.7945$

$e\hat{\ }(-0.23)$



4.3 Logarithmic Functions

Inverses of Exponential Functions

$f(x) = 2^x$

$y = 2^x$

$x = 2^y$

y = the power to which we raise 2 to get x

$f^{-1}(x) =$ ___ " ___

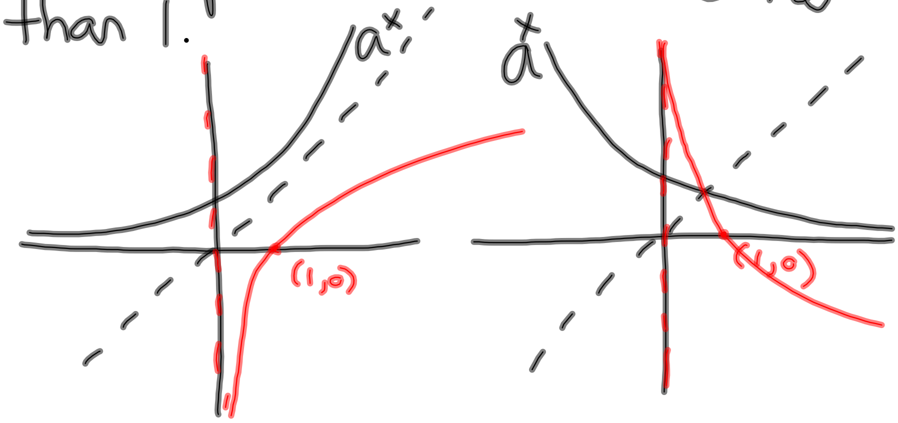
$f^{-1}(x) = \log_2 x$ "log, base 2, of x"

$\log_2 8 = 3 \iff 2^3 = 8$

3 is the power to which we raise 2 to get 8

For $f(x)=a^x$, the inverse function is $f^{-1}(x)=\log_a x$.

$y=\log_a x$ is the number y such that $x=a^y$, where $x>0$ and a is a positive constant other than 1.



$a>1$ $\qquad$ $0<a<1$

Simplify/evaluate:

$\log_{10} 1000 = 3$ $\qquad$ $10^{\boxed{3}} = 1000$

$\log_{10} 0.001 = -3$ $\qquad$ $10^{\boxed{-3}} = \frac{1}{1000}$

$\log_3 27 = 3$

$\log_5 1 = 0$

$\log_6 6 = 1$

$\log_a 1 = 0$
&
$\log_a a = 1$
for any base a

$$\log_a x = y \longleftrightarrow a^y = x$$

$$32 = 2^x \longleftrightarrow \log_2 32 = x$$

$$\log_2 64 = x \longleftrightarrow 64 = 2^x$$

$\boxed{\log}$ on your calculator is $\log_{10} x$

"common log"

$\boxed{\ln}$ is $\log_e x$

"natural log"

<u>4.2</u> # 5, 7, 9, 31, 45

<u>4.3</u> # 9-27 odd

Quiz moved to Thurs.