4.2 Exponential Functions

$$f(x) = a^{x}$$

base exponent

$a > 0$ (so we don't have things like $(-1)^{1/2}$)

$a \neq 1$ (exclude constant function 1^x)

Note: variable is in exponent not base, unlike in power functions

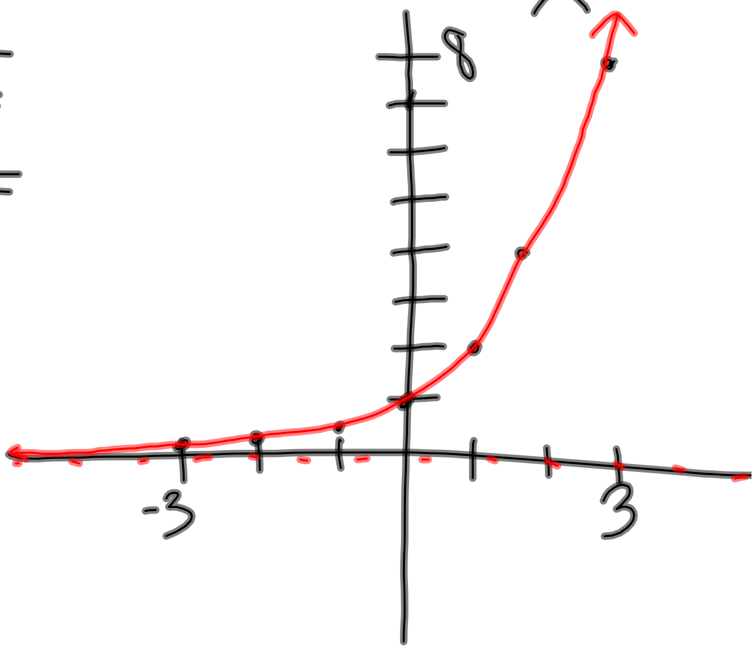
$f(x) = x^2$ v. $f(x) = \left(\frac{1}{3}\right)^x$

power fn/polyn. exp.

$$f(x) = 2^x$$

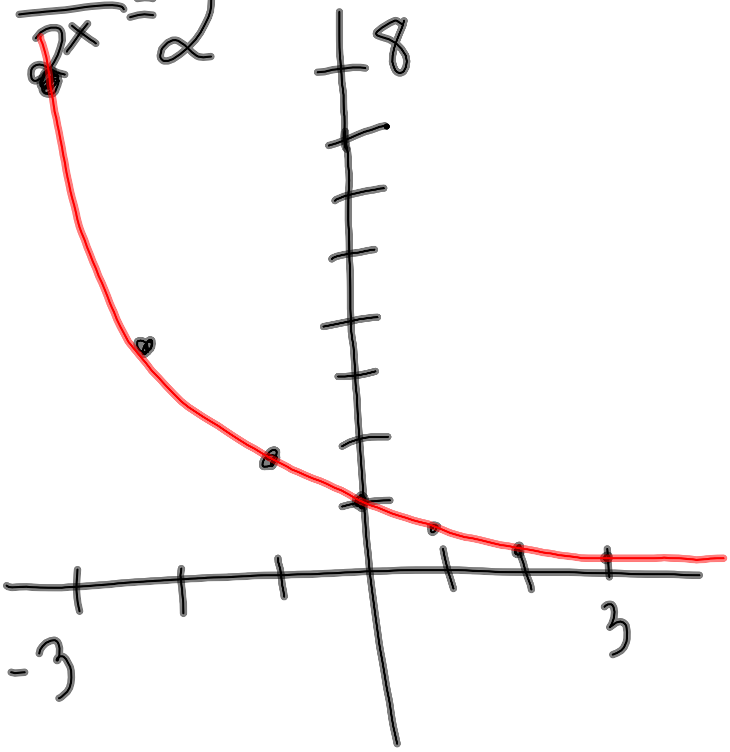
$$x^{-n} = \frac{1}{x^n}$$

x	$f(x)=2^x$
-3	$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$
-2	$2^{-2} = \frac{1}{2^2} = \frac{1}{4}$
-1	$2^{-1} = \frac{1}{2^1} = \frac{1}{2}$
0	$2^0 = 1$
1	$2^1 = 2$
2	$2^2 = 4$
3	$2^3 = 8$



$$f(x) = \left(\frac{1}{2}\right)^x = \frac{1}{2^x} = 2^{-x}$$

x	$\left(\frac{1}{2}\right)^x = \frac{1}{2^x}$
-3	$\frac{1}{2^{-3}} = 2^3 = 8$
-2	$\frac{1}{2^{-2}} = 2^2 = 4$
-1	$\frac{1}{2^{-1}} = 2^1 = 2$
0	$\frac{1}{2^0} = \frac{1}{1} = 1$
1	$\frac{1}{2^1} = \frac{1}{2}$
2	$\frac{1}{2^2} = \frac{1}{4}$
3	$\frac{1}{2^3} = \frac{1}{8}$



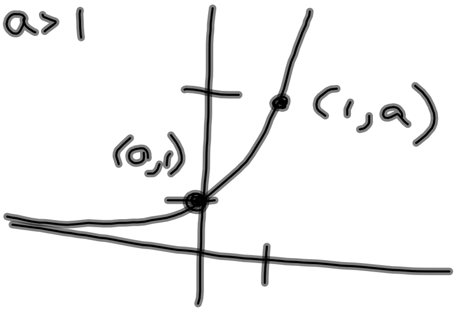
Properties of Exponential Functions

$f(x) = a^x$, $a > 0$, $a \neq 1$

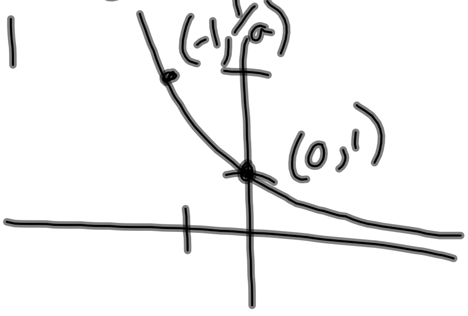
- continuous
- one-to-one
- Domain: $(-\infty, \infty)$
- Range: $(0, \infty)$

- Horizontal asymptote: $y = 0$
- y-intercept: $(0, 1)$
- increasing if $a > 1$
- decreasing if $0 < a < 1$

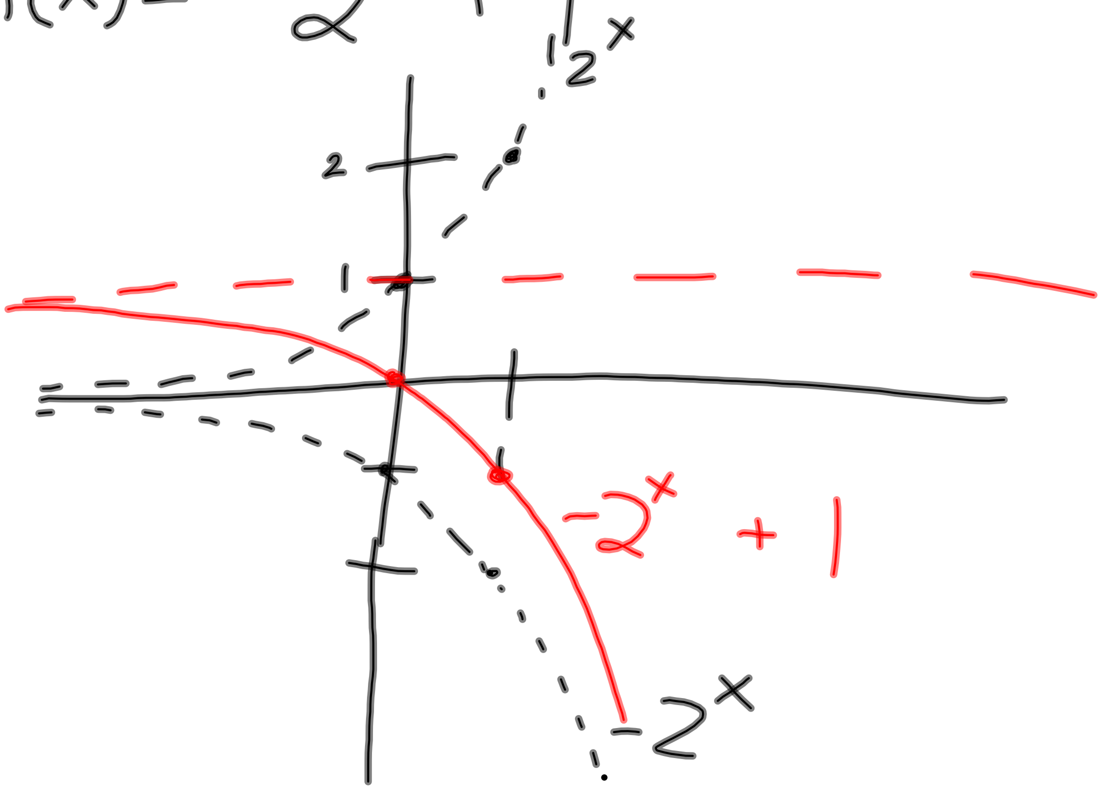
$a > 1$



$0 < a < 1$



$f(x) = -2^x + 1$



Application: Compound Interest

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

A = amount of money

P = principal (initial investment)

t = time in # of years

r = interest rate (decimal)

n = # of times interest is compounded per year

Example:

$100 @ 5% interest compounded quarterly for 1 year

$$A(t) = P\left(1+\frac{r}{n}\right)^{nt}$$

$$= 100\left(1+\frac{0.05}{4}\right)^{4\cdot 1}$$

$$= \$105.09$$

If we invest \$1 at 100% interest for 1 year,

$$A=P\left(1+\frac{r}{n}\right)^{nt}=1\left(1+\frac{1}{n}\right)^{n}$$

$$A=\left(1+\frac{1}{n}\right)^{n}$$

n	$A(n)=\left(1+\frac{1}{n}\right)^{n}$
1 annually	$\left(1+\frac{1}{1}\right)^{1}=\$2$
2 semiannually	$\left(1+\frac{1}{2}\right)^{2}=\$2.25$
4 quarterly	$\left(1+\frac{1}{4}\right)^{4}=2.4414$
12 montly	$\left(1+\frac{1}{12}\right)^{12}=2.6130$
365 daily	$\left(1+\frac{1}{365}\right)^{365}=2.7146$
8760 hourly	$\left(1+\frac{1}{8760}\right)^{8760}=2.7181$
$\to\infty$	$e\approx 2.7182818284\ldots$

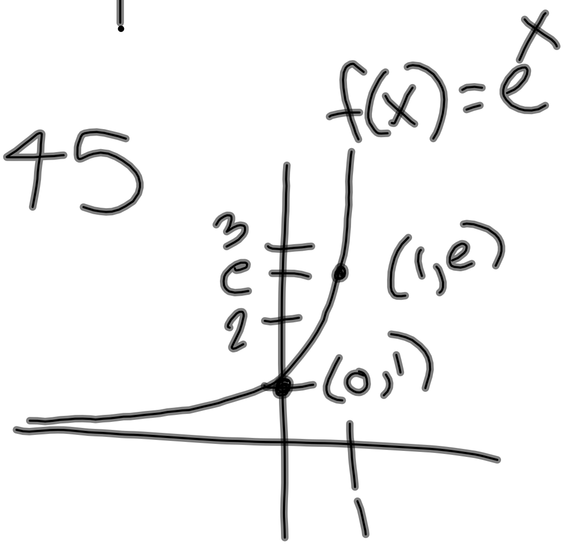
(e is an irrational # like π)

$$e=2.718$$

$$e^{2}=e\hat{\ }2\approx 7.389$$

$$e^{-0.23}\approx 0.7945$$

$$e\hat{\ }(-0.23)$$



4.3 Logarithmic Functions

Inverses of Exponential Functions

$f(x) = 2^x$

$y = 2^x$

$x = 2^y$

y = the power to which we raise 2 to get x

$f^{-1}(x) =$ ______ " _________

"log, base 2, of x"

$f^{-1}(x) = \log_2 x$

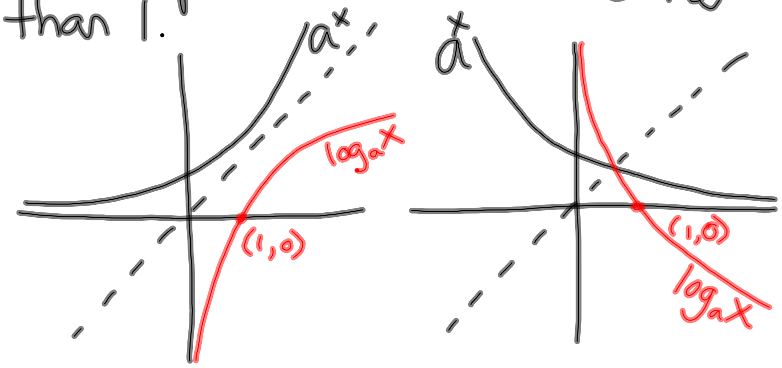
$$\log_2 8 = 3 \iff 2^3 = 8$$

↑

3 is the power to which we raise 2 to get 8

For $f(x)=a^x$, the inverse function is $f^{-1}(x)=\log_a x$.

$y=\log_a x$ is the number y such that $x=a^y$, where $x>0$ and a is a positive constant other than 1.



$a>1$ $0<a<1$

$\log_a x$: domain $(0,\infty)$ x-int: $(1,0)$
range: $(-\infty,\infty)$ V.A.: $x=0$

Simplify/evaluate:

$\log_{10} 1000 = 3$ $(10^3=1000)$

$\log_{10} 0.001 = -3$ $\left(10^{-3}=\frac{1}{10^3}=\frac{1}{1000}=0.001\right)$

$\log_3 27 = 3$

$\log_5 1 = 0$

$\log_6 6 = 1$

$\log_a 1 = 0$
$\log_a a = 1$
for any log base a

$$\log_a x = y \longleftrightarrow a^y = x$$

$$32 = 2^x \longleftrightarrow \log_2 32 = x$$

$$\log_2 64 = x \longleftrightarrow 2^x = 64$$

$\boxed{\text{log}}$ on your calculator is $\log_{10} x$ "common log"

$\boxed{\text{ln}}$ is $\log_e x$ "natural log"

4.2 # 5, 7, 9, 31, 45

4.3 # 9-27 odd

Quiz moved to Thurs.