

Review:

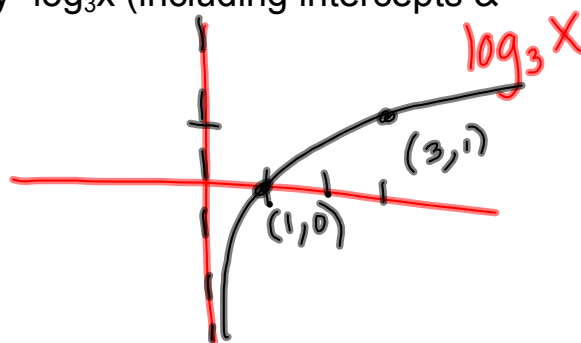
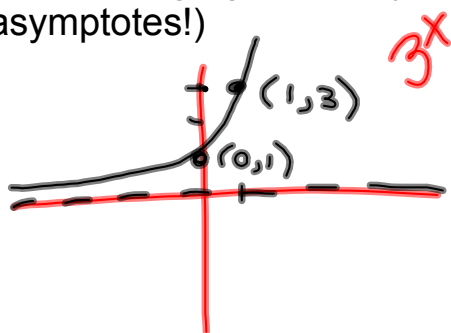
In the expression $\log_a b = c$,
c is the power to which we raise a to get b. $a^c = b$

Evaluate the following:

$$\log_2 16 = 4 \quad \log_2(1/4) = -2 \quad \log(100) = \log_{10} 100 = 2$$

$$\log(1/10) = -1 \quad \ln(e) = \log_e e = 1 \quad \ln(1) = \log_e(1) = 0$$

Sketch rough graphs of $y=3^x$ and $y=\log_3 x$ (including intercepts & asymptotes!)

Change of Base Formula

$$\log_b M = \frac{\log_a M}{\log_a b}$$

$$\log_6 7 = \frac{\log 7}{\log 6} = \frac{\ln 7}{\ln 6} \approx 1.09$$

$$(\ln(7)) / (\ln(6))$$

$$\log_8 32 = \frac{\log_2 32}{\log_2 8} = \frac{5}{3}$$

4.4 Properties of Logarithmic Functions

Review-Properties of Exponents

$$a^m a^n = a^{m+n}$$

$$(a^m)^n = a^{mn}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

Product Rule

$$\log_a(M \cdot N) = \log_a M + \log_a N$$

$M, N > 0$

Power Rule

$$\begin{aligned}\log_a(M^p) &= \log_a(\overbrace{M \cdot M \cdots M}^{p \text{ times}}) \\ &= \underbrace{\log_a M + \log_a M + \cdots + \log_a M}_{p \text{ times}} \\ &= p \log_a M \\ \log_a M^p &= p \log_a M\end{aligned}$$

Quotient Rule

$$\log_a \frac{M}{N} = \log_a M - \log_a N$$

common errors

$$\log_a MN \neq (\log_a M)(\log_a N)$$

$$\log_a(M+N) \neq \log_a M + \log_a N$$

$$\log_a \frac{M}{N} \neq \frac{\log_a M}{\log_a N}$$

$$(\log_a M)^p \neq p \log_a M$$

Inverse Properties

$$\log_a a^x = x \quad ; \quad a^{\log_a x} = x$$

Examples

$$2. \log_2(8 \cdot 64) = \log_2 8 + \log_2 64$$

$$= 3 + 6 = \boxed{9}$$

$$10. \log_a X^4 = \boxed{4 \log_a X}$$

$$16. \ln \sqrt{a} = \ln (a)^{1/2} = \boxed{\frac{1}{2} \ln a}$$

$\sqrt[n]{X} = X^{1/n}$

$$22. \log_b \frac{3}{w} = \boxed{\log_b 3 - \log_b w}$$

$$\begin{aligned}
 32. \log_c \sqrt[3]{\frac{y^3 z^2}{x^4}} &= \log_c \left(\frac{y^3 z^2}{x^4} \right)^{1/3} \\
 &= \frac{1}{3} \log_c \left(\frac{y^3 z^2}{x^4} \right) = \\
 &= \frac{1}{3} \left[\log_c (y^3 z^2) - \log_c (x^4) \right] \\
 &= \frac{1}{3} \left[\log_c y^3 + \log_c z^2 - \log_c x^4 \right] \\
 &= \frac{1}{3} \left[3 \log_c y + 2 \log_c z - 4 \log_c x \right] \\
 &= \boxed{\log_c y + \frac{2}{3} \log_c z - \frac{4}{3} \log_c x}
 \end{aligned}$$

$$\begin{aligned}
 36. \log 0.01 + \log 1000 \\
 &= \log (0.01 \cdot 1000) = \log 10 = \boxed{1}
 \end{aligned}$$

$$\begin{aligned}
 44. \ln 2x + 3(\ln x - \ln y) \\
 &= \ln 2x + 3 \ln x - 3 \ln y \\
 &= \ln 2x + \ln x^3 - \ln y^3 \\
 &= \ln 2x + \ln \frac{x^3}{y^3} \\
 &= \ln \frac{2x \cdot x^3}{y^3} \\
 &= \boxed{\ln \frac{2x^4}{y^3}}
 \end{aligned}$$

$$50. \frac{2}{3} [\ln(x^2-9) - \ln(x+3)] + \ln(x+y)$$

rewrite as a single logarithm

$$= \frac{2}{3} \left[\ln \left(\frac{x^2-9}{x+3} \right) \right] + \ln(x+y)$$

$$= \frac{2}{3} \left[\ln \frac{\cancel{x+3}(x-3)}{\cancel{x+3}} \right] + \ln(x+y)$$

$$= \frac{2}{3} \ln(x-3) + \ln(x+y)$$

$$= \ln(x-3)^{2/3} + \ln(x+y)$$

$$= \ln \left[(x-3)^{2/3} (x+y) \right]$$

$$\log_a 2 \approx 0.301, \log_a 7 \approx 0.845, \log_a 11 \approx 1.041$$

$$54. \log_a 14 = \log_a (7 \cdot 2) = \log_a 7 + \log_a 2$$

$$= 0.845 + 0.301$$

$$= \boxed{1.146}$$

$$56. \log_a \frac{1}{7} = \log_a 1 - \log_a 7$$

$$= 0 - 0.845 = -0.845$$

$$58. \log_a 9 = \log_a (11-2)$$

insufficient
info. given

4.2 # 21, 33, 35, 43

4.3 # 29-53 odd, 71-77 odd,
83-89 odd

& memorize log rules!