Review:

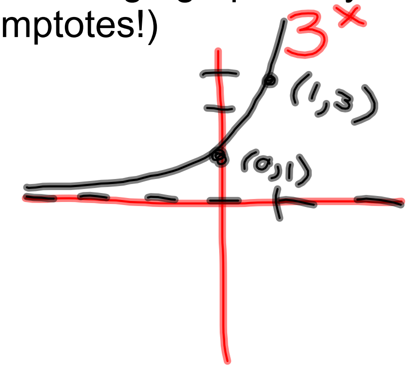
In the expression $\log_a b=c$,
$\underline{c}$ is the power to which we raise $\underline{a}$ to get $\underline{b}$. $a^c=b$

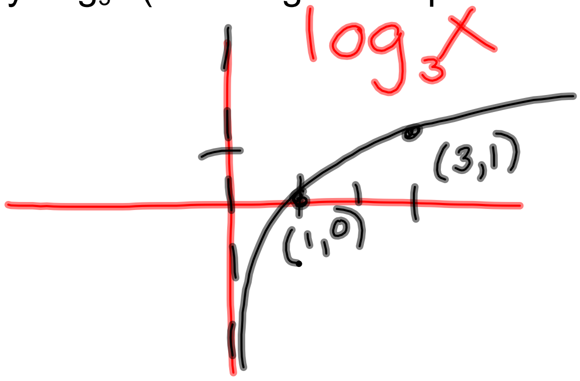
Evaluate the following:

$\log_2 16=4$ $\quad$ $\log_2(1/4)=-2$ $\quad$ $\log(100)=\log_{10}100=2$

$\log(1/10)=-1$ $\quad$ $\ln(e)=\log_e e=1$ $\quad$ $\ln(1)=\log_e 1=0$

Sketch rough graphs of $y=3^x$ and $y=\log_3 x$ (including intercepts & asymptotes!)





Change of Base Formula

$$\log_b M=\frac{\log_a M}{\log_a b}$$

$$\log_6 7=\frac{\log 7}{\log 6}=\frac{\ln 7}{\ln 6}\approx 1.09$$

(ln(7))/((ln(6))

$$\log_8 32 = \frac{\log_2 32}{\log_2 8} = \boxed{\frac{5}{3}}$$

4.4 Properties of Logarithmic Functions

Review-Properties of Exponents

$$a^m a^n = a^{m+n}$$

$$(a^m)^n = a^{mn}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

Product Rule

$$\log_a(M \cdot N) = \log_a M + \log_a N$$

$M, N > 0$

Power Rule

$$\log_a(M^p) = \log_a(\overbrace{M \cdot M \cdots M}^{p \text{ times}})$$

$$= \underbrace{\log_a M + \log_a M + \cdots + \log_a M}_{p \text{ times}}$$

$$= p \log_a M$$

$$\log_a M^p = p \log_a M$$

Quotient Rule

$$\log_a \frac{M}{N} = \log_a M - \log_a N$$

common **errors**

$$\log_a MN \neq (\log_a M)(\log_a N)$$

$$\log_a (M+N) \neq \log_a M + \log_a N$$

$$\log_a \frac{M}{N} \neq \frac{\log_a M}{\log_a N}$$

$$(\log_a M)^p \neq p\log_a M$$

Inverse Properties

$$\log_a a^x = x \quad ; \quad a^{\log_a x} = x$$

Examples

2. $\log_2(8\cdot 64) = \log_2 8 + \log_2 64 = 3+6 = \boxed{9}$

10. $\log_a x^4 = \boxed{4\log_a x}$

16. $\ln\sqrt{a} = \ln a^{\frac{1}{2}}$ $\qquad \sqrt[n]{x} = x^{1/n}$

$= \boxed{\frac{1}{2}\ln a}$

22. $\log_b \frac{3}{w} = \boxed{\log_b 3 - \log_b w}$

32. $\log_c \sqrt[3]{\dfrac{y^3z^2}{x^4}} = \log_c \left(\dfrac{y^3z^2}{x^4}\right)^{\frac{1}{3}}$

$= \frac{1}{3}\log_c \left(\dfrac{(y^3z^2)}{(x^4)}\right) = \frac{1}{3}\left[\log_c(y^3z^2) - \log_c x^4\right]$

$= \frac{1}{3}\left[\log_c y^3 + \log_c z^2 - \log_c x^4\right]$

$= \frac{1}{3}\left[3\log_c y + 2\log_c z - 4\log_c x\right]$

$= \boxed{\log_c y + \frac{2}{3}\log_c z - \frac{4}{3}\log_c x}$

36. $\log 0.01 + \log 1000$

$= \log(0.01 \cdot 1000) = \log 10 = \boxed{1}$

44. $\ln 2x + 3(\ln x - \ln y)$

$= \ln 2x + 3\ln x - 3\ln y$

$= (\ln 2x + \ln x^3) - \ln y^3$

$= \ln(2x \cdot x^3) - \ln y^3$

$= \boxed{\ln\left(\dfrac{2x^4}{y^3}\right)}$

50. $\frac{2}{3}\left[\ln(x^2-9)-\ln(x+3)\right]+\ln(x+y)$

rewrite as a single logarithm

$= \frac{2}{3}\left[\ln\frac{x^2-9}{x+3}\right]+\ln(x+y)$

$= \frac{2}{3}\ln\frac{(x\cancel{+3})(x-3)}{x\cancel{+3}}+\ln(x+y)$

$= \frac{2}{3}\ln(x-3)+\ln(x+y)$

$= \ln(x-3)^{2/3}+\ln(x+y)$

$= \boxed{\ln\left[(x-3)^{2/3}(x+y)\right]}$

$\log_a 2 \approx 0.301, \log_a 7 \approx 0.845, \log_a 11 \approx 1.041$

54. $\log_a 14 = \log_a(2\cdot 7) = \log_a 2 + \log_a 7$

$= 0.301 + 0.845$

$= \boxed{1.146}$

56. $\log_a \frac{1}{7} = \log_a 1 - \log_a 7 =$

$= 0 - 0.845 = \boxed{-0.845}$

58. $\log_a 9 = \log_a(2+7) = \log_a(11-2$

insufficient information to solve

4.2 #21, 33, 35, 43

4.3 #29-53 odd, 71-77 odd,
83-89 odd

& memorize log rules!