Review: **Inverse Functions**

A function is one-to-one if $f(a) = f(b)$ implies that $a = b$ for all a, b in the domain of f. That is, in addition to being a function (each x maps to exactly one y), a one-to-one function only has one x-value mapping to each y-value.
The graph of a one-to-one function passes both the horizontal and vertical line tests.

Example of verifying that a function is one-to-one:

$f(x) = (x+4)^3 - 5$
$f(a) = f(b)$
$(a+4)^3 - 5 = (b+4)^3 - 5$
$(a+4)^3 = (b+4)^3$
$\sqrt[3]{(a+4)^3} = \sqrt[3]{(b+4)^3}$
$(a + 4) = (b + 4)$
$a = b$
Since $f(a) = f(b)$ implies that $a = b$, f is one-to-one.

Example showing that a function is NOT one-to-one:

$f(x) = x^2$
$f(2) = 4$
$f(-2) = 4$
Since two different x-values yield the same y-value, f is not one-to-one.

The one-to-one functions $f(x)$ and $g(x)$ are inverses if:
$(f \circ g)(x) = x \text{ and } (g \circ f)(x) = x$

Example of verifying that two functions are inverses:

$f(x) = x^3 \quad g(x) = \sqrt[3]{x}$
$(f \circ g)(x) = \left(\sqrt[3]{x}\right)^3 = x$
$(g \circ f)(x) = \sqrt[3]{(x^3)} = x$

How to find an inverse function:

1. Replace $f(x)$ with y
2. Interchange x and y
3. Solve for y in terms of x.
4. Replace y with $f^{-1}(x)$.

Example of finding an inverse function:

$$f(x) = \frac{5x-3}{2x+1}$$
$$y = \frac{5x-3}{2x+1}$$
$$x = \frac{5y-3}{2y+1}$$
$x(2y+1) = 5y - 3$
$2xy + x = 5y - 3$
$x + 3 = 5y - 2xy$
$x + 3 = y(5 - 2x)$
$$y = \frac{3+x}{5-2x}$$
$$\boxed{f^{-1}(x) = \frac{3+x}{5-2x}}$$

Properties of Exponential Functions:

$a^m a^n = a^{m+n}$
$\frac{a^m}{a^n} = a^{m-n} = \frac{1}{a^{n-m}}$
$(a^m)^n = a^{mn}$
$(a^p b^q)^r = a^{pr} b^{qr}$, $\left(\frac{a^p}{b^q}\right)^r = \frac{a^{pr}}{b^{qr}}$
$a^{-1} = \frac{1}{a}$
$a^0 = 1, a \neq 0$

Properties of Logarithmic Functions:

Product Rule: $log_a(MN) = log_a M + log_a N$
Power Rule: $log_a(M)^P = p\, log_a M$
Quotient Rule: $log_a\left(\frac{M}{N}\right) = log_a M - log_a N$
Change of Base Formula: $log_b M = \frac{log_a M}{log_a b}$
Other Properties: $log_a a = 1 \quad log_a 1 = 0$
$log_a a^x = x \quad a^{log_a x} = x$
$\ln x = \log_e x \quad \log x = \log_{10} x$

The logarithmic equation $\log_a b = c$ is equivalent to the exponential equation $a^c = b$

Compound Interest

The amount of money A that a principal P will grow to after t years at interest rate r (in decimal form), compounded times per year, is given by the formula

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Express in terms of sums and differences of logarithms.

24. $\log_a x^3y^2z$ $= \log_a x^3 + \log_a y^2 + \log_a z$

$= \boxed{3\log_a x + 2\log_a y + \log_a z}$

26. $\log_b \dfrac{x^2y}{b^3}$ $= \log_b x^2 + \log_b y - \log_b b^3$

$= \boxed{2\log_b x + \log_b y - 3}$

Express in terms of sums and differences of logarithms.

30. $\ln \sqrt[3]{5x^5}$ $= \ln(5x^5)^{1/3} = \ln(5^{1/3}x^{5/3})$

$= \ln 5^{1/3} + \ln x^{5/3}$

$= \boxed{\frac{1}{3}\ln 5 + \frac{5}{3}\ln x}$

34. $\log_a \sqrt{\dfrac{a^6b^8}{a^2b^5}}$ $= \log_a \sqrt{a^{6-2}b^{8-5}} = \log_a \sqrt{a^4b^3}$

$= \log_a (a^4b^3)^{1/2} = \log_a (a^2b^{3/2}) =$

$= \log_a a^2 + \log_a b^{3/2} = \boxed{2 + \frac{3}{2}\log_a b}$

Express as a single logarithm and, if possible, simplify.

40. $\frac{1}{2}\log a - \log 2$ $= \log a^{1/2} - \log 2 = \log\left(\frac{a^{1/2}}{2}\right)$

$= \boxed{\log\left(\frac{\sqrt{a}}{2}\right)}$

48. $\log_a \frac{a}{\sqrt{x}} - \log_a \sqrt{ax}$ $= \log_a\left(\frac{\frac{a}{\sqrt{x}}}{\sqrt{ax}}\right)$

$= \log_a\left(\frac{a}{\sqrt{x}} \cdot \frac{1}{\sqrt{ax}}\right) = \log_a\left(\frac{a}{\sqrt{a}\sqrt{x^2}}\right)$

$= \log_a\left(\frac{a}{x\sqrt{a}}\right) = \log_a\left(\frac{\sqrt{a}}{x}\right)$

$\frac{a}{\sqrt{a}} \cdot \frac{\sqrt{a}}{\sqrt{a}} = \frac{\cancel{a}\sqrt{a}}{\cancel{a}} = \sqrt{a}$

$= \log_a a^{1/2} - \log_a x$

$= \boxed{\frac{1}{2} - \log_a x}$

Express as a single logarithm and, if possible, simplify.

52. $120\left(\ln\sqrt[5]{x^3} + \ln\sqrt[3]{y^2} - \ln\sqrt[4]{16z^5}\right)$

$= 120\ln\left(\frac{\sqrt[5]{x^3}\cdot\sqrt[3]{y^2}}{\sqrt[4]{16z^5}}\right)$

$= \ln\left(\frac{x^{3/5}y^{2/3}}{16^{1/4}z^{5/4}}\right)^{120}$

$= \ln\left(\frac{x^{\frac{3}{\cancel{5}}\cdot\frac{\cancel{120}^{24}}{1}}\, y^{\frac{2}{\cancel{3}}\cdot\frac{\cancel{120}^{40}}{1}}}{(2^{\cancel{4}})^{\frac{1}{\cancel{4}}\cdot\frac{120}{1}}\, z^{\frac{5}{\cancel{4}}\cdot\frac{\cancel{120}^{30}}{1}}}\right)$

$= \boxed{\ln\left(\frac{x^{72}y^{80}}{2^{120}z^{150}}\right)}$

Simplify.

$\log_a b = c \Leftrightarrow a^c = b$

66. $\log_t t^{2713}$ $= 2713$

68. $\log_q q^{\sqrt{3}}$ $= \sqrt{3}$

$\underbrace{5^x \ \& \ \log_5 x}_{\text{inverses}}$

70. $5^{\log_5(4x-3)}$ $= 4x-3$

72. $e^{\ln x^3}$ $= x^3$

74. $\log 10^{-k}$ $= -k$

76. $\log_b \sqrt{b^3}$ $= \log_b (b^3)^{1/2} = \log_b b^{3/2} = \boxed{\frac{3}{2}}$

Determine whether the statement is true. Assume that a, x, M, and N are positive.

102. $\log_N (MN)^x = x \log_N M + x$

$LHS = x \log_N (MN) =$

$= x[\log_N M + \log_N N] =$

$= x[\log_N M + 1] =$

$= x \log_N M + x = RHS.$

Write without using logarithms.

106. $\log_a x + \log_a y - mz = 0$

$\log_a b = c \Leftrightarrow a^c = b$

$\log_a x + \log_a y = mz$

$\log_a xy = mz$

$\boxed{a^{mz} = xy}$

Homework:

4.4 # 31,33, 49-55odd; 65-75 odd; 107