4. $\log_2 16 =$ 4

5. $\log_3\left(\frac{1}{9}\right) =$ -2 $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

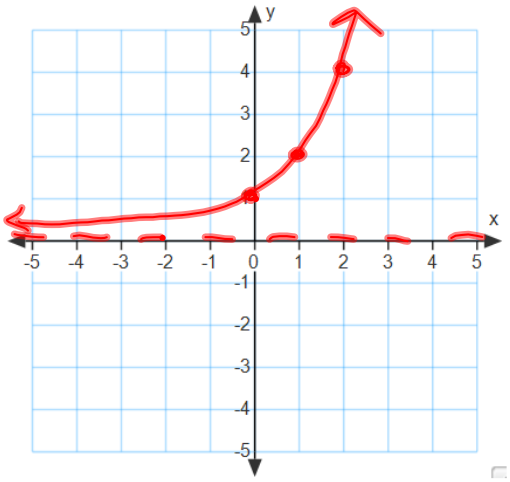
6. $\log 1000 =$ 3

7. $\log 1 =$ 0

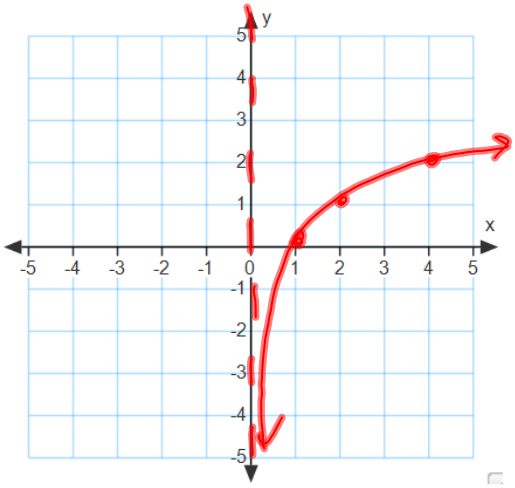
8. $\ln e =$ 1 $\log_e e$ $e^1 = e$

9. $\ln 1 =$ 0 $\log_e 1$ $e^0 = 1$

10. $y = 2^x$



11. $y = \log_2 x$



4.4

107. $\ln a - \ln b + xy = 0$

$\ln a - \ln b = -xy$

$\ln \frac{a}{b} = -xy$

$\log_e \frac{a}{b} = -xy$

$\log_a b = c$

$a^c = b$

$e^{-xy} = \frac{a}{b}$

$\frac{1}{e^{xy}} = \frac{a}{b}$

$e^{xy} = \frac{b}{a}$

49. $\ln x - 3[\ln(x-5) + \ln(x+5)]$

$\ln x - 3\ln((x-5)(x+5))$

$\ln x - \ln(x^2-25)^3$

$\ln \frac{x}{(x^2-25)^3}$

4.5 Solving Exponential & Logarithmic Equations

$3^{2x} = 3^5$

$2x = 5$

$x = \frac{5}{2}$

$\log_3 2x = \log_3 5$

$2x = 5$

$x = 5/2$

For any $a>0$, $a\neq 1$,

$$a^x = a^y \iff x = y$$

Similarly, for $M, N>0$, $a>0$, $a\neq 1$,

$$\log_a M = \log_a N \iff M = N$$

$$e^{50t} = 300$$

$$\ln e^{50t} = \ln 300$$

$$50t = \ln 300$$

$$t = \frac{\ln 300}{50} \approx 0.114$$

$\log x + \log(x+3) = 1$

$\log_{10}(x(x+3)) = 1$

$10^1 = x(x+3)$

$0 = x^2+3x-10$

$0 = (x+5)(x-2)$

$x = 2, -5$

* always check answers for log equations!

Domain of $\log_a x$ is $(0,\infty)$!

4. $3^{7x} = 27$

$3^{7x} = 3^3$

$7x = 3$

$x = \frac{3}{7}$

10. $3^{x^2+4x} = \frac{1}{27}$

$3^{x^2+4x} = 3^{-3}$

$x^2+4x = -3$

$x^2+4x+3 = 0$

$(x+3)(x+1) = 0$

$x = -1, -3$

12. $28^x = 10^{-3x}$

$\log 28^x = \log 10^{-3x}$

$x \log 28 = -3x \cancel{\log 10}^{1}$

$x \log 28 = -3x$

$x \log 28 + 3x = 0$

$x(\log 28 + 3) = 0$

$\boxed{x = 0}$

20. $5^{x+2} = 4^{1-x}$

$\ln 5^{x+2} = \ln 4^{1-x}$

$(x+2)\ln 5 = (1-x)\ln 4$

$x \ln 5 + 2\ln 5 = \ln 4 - x \ln 4$

$x \ln 5 + x \ln 4 = \ln 4 - 2\ln 5$

$x(\ln 5 + \ln 4) = \ln 4 - 2\ln 5$

$$\boxed{x = \frac{\ln 4 - 2\ln 5}{\ln 5 + \ln 4} = \frac{\ln\left(\frac{4}{25}\right)}{\ln 20} \approx -0.6}$$

24. $(e^x - 6e^{-x} = 1) \cdot e^x$

$e^x e^x - 6e^{-x} e^x = e^x$

$e^{2x} - 6 = e^x$

$e^{2x} - e^x - 6 = 0$

$(e^x)^2 - e^x - 6 = 0$

$u = e^x$

$u^2 - u - 6 = 0$

$(u-3)(u+2) = 0$

$u = 3 \quad u = -2$

$e^x = 3 \quad e^x = -2$

$\ln e^x = \ln 3 \quad , \ln e^x = \ln(-2)$

$x = \ln 3 \quad , x = \ln(-2)$

$a^m a^n = a^{m+n}$

$e^{-x} e^x = e^{-x+x} = e^0 = 1$

26. $\left(\frac{5^x - 5^{-x}}{5^x + 5^{-x}} = 8\right) \cdot (5^x + 5^{-x})$

$5^x - 5^{-x} = 8(5^x + 5^{-x})$

$1 \cdot 5^x - 1 \cdot 5^{-x} = 8 \cdot 5^x + 8 \cdot 5^{-x}$

$0 = 8 \cdot 5^x - 1 \cdot 5^x + 8 \cdot 5^{-x} + 1 \cdot 5^{-x}$

$(0) = (7 \cdot 5^x + 9 \cdot 5^{-x}) \cdot 5^x$

$0 = 7 \cdot 5^{2x} + 9$

$-9 = 7 \cdot 5^{2x}$

$\frac{-9}{7} = 5^{2x}$

$\ln\left(\frac{-9}{7}\right) = 2x \ln 5$

no solution :(

Homework:

4.5 # 1-25 odd