4.5

13. $e^{-c} = 5^{2c}$

$\ln e^{-c} = \ln 5^{2c}$

$-c = 2c(\ln 5)$

$0 = (2\ln 5)c + c$

$0 = c(2\ln 5 + 1)$

$\boxed{0 = c}$

25. $\frac{e^{x}+e^{-x}}{(e^{x}-e^{-x})} = 3$

$e^{x}+e^{-x} = 3(e^{x}-e^{-x})$

$e^{x}+e^{-x} = 3e^{x}-3e^{-x}$

$0 = 3e^{x}-e^{x}-3e^{-x}-e^{-x}$

$e^{x} \cdot 0 = (2e^{x}-4e^{-x})e^{x}$

$0 = 2e^{x}e^{x}-4e^{-x}e^{x}$ $e^{-x+x} = e^{0} = 1$

$0 = 2e^{2x}-4$

$4 = 2e^{2x}$

$2 = e^{2x}$

$\ln 2 = 2x$

$\boxed{x = \frac{\ln 2}{2}} \approx 0.347$

34. $\log_5(8-7x)=3$

$\log_a b = c \Leftrightarrow a^c = b$

$5^3 = 8-7x$

$125 = 8-7x$

$117 = -7x$

$-\frac{117}{7} = x$

$8-7\left(-\frac{117}{7}\right)$

$= 8+117$

38. $\log(x+5)-\log(x-3)=\log 2$

$\log\frac{x+5}{x-3}=\log 2$

$\log_a M = \log_a N \Rightarrow M=N$

$\frac{x+5}{x-3}=2$

$x+5=2(x-3)$

$x+5=2x-6$

$\boxed{11=x}$

42. $\ln x - \ln(x-4) = \ln 3$

$\ln \frac{x}{x-4} = \ln 3$

$\frac{x}{x-4} = 3$

$x = 3(x-4)$

$x = 3x - 12$

$12 = 3x - x$

$12 = 2x$

$\boxed{6 = x}$

46. $\log_5(x+4) + \log_5(x-4) = 2$

$\log_5\left[\underbrace{(x+4)(x-4)}_{x^2-16}\right] = 2$

$5^2 = x^2 - 16$

$25 + 16 = x^2$

$41 = x^2$

$\cancel{\pm}\sqrt{41} = x$

$\sqrt{36} < \sqrt{41} < \sqrt{49}$

$6 < \sqrt{41} < 7$

$\boxed{x = \sqrt{41}}$

48. $\log_3 x + \log_3(x+1) = \log_3 2 + \log_3(x+3)$

$\log_3[x(x+1)] = \log_3[2(x+3)]$

$x^2 + x = 2x + 6$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$\boxed{x=3}$, $\cancel{x=-2}$

4.6 Applications & Models

Population Growth

$P(t) = P_0 e^{kt}$, $k>0$

$P(t)$ = population at time t

P_0 = initial population

k = exponential growth rate

Continuously Compounded Interest

$P(t) = P_0 e^{kt}$

$P(t)$ = amt of $ @ time t

P_0 = initial investment

k = interest rate

How to find Growth Rate / Doubling Time

$P(t) = P_0 e^{kt}$

$\log_a b^p = p \log_a b$

$\frac{P(t)}{P_0} = e^{kt}$

$\ln\left(\frac{P(t)}{P_0}\right) = kt$

$2P_0 = P_0 e^{kt}$

$2 = e^{kt}$

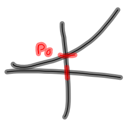
$\boxed{\ln 2 = kt}$ doubling time

2. exponential growth rate of rabbits is 11.7% per day ($0.117 = k$)
initial population of 100 rabbits (P_0)

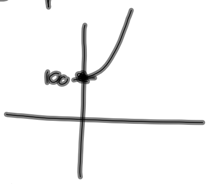
a. Find the exponential Growth Function

$P(t) = P_0 e^{kt}$

$\boxed{P(t) = 100e^{0.117t}}$



b. Graph the function



c. What will the population be after 7 days?

$P(7) = 100e^{0.117(7)} \approx \boxed{227 \text{ rabbits}}$

d. Find the Doubling Time.

$2P_0 = P_0 e^{kt}$

$2 = e^{kt}$

$\ln 2 = kt$

$t = \frac{\ln 2}{k} = \frac{\ln 2}{0.117}$

$\approx \boxed{5.9 \text{ days}}$

8. Interest Compounded Continuously $\boxed{P(t) = P_0 e^{kt}}$

Initial Investment P_0	Interest Rate k	Doubling Time T	Amount after 5 years $P(5)$
a) \$35,000	6.2%	$2P_0 = P_0 e^{0.062t}$ $\ln 2 = 0.062t$ $t = 11.2$ yr	$P(5) = 35{,}000 \cdot e^{0.062 \cdot 5}$ $= 47{,}719.88$
b) \$5,000	$7130.90 = 5000e^{k \cdot 5}$ $\ln\left(\frac{7130.90}{5000}\right) = k \cdot 5$ $k = 7.1\%$	$t = \frac{\ln 2}{k} = \frac{\ln 2}{0.071}$ $t = 9.8$ yr	\$7,130.90
c) $11414.71 = P_0 e^{0.084 \cdot 5}$ $P_0 = \frac{11414.71}{e^{0.084 \cdot 5}}$ $= \$7500$	8.4%	$t = \frac{\ln 2}{0.084} =$ $= 8.3$ yr	\$11,414.71
d) $P_0 = \frac{17539.32}{e^{0.063 \cdot 5}}$ $= \$12{,}800$	$11 = \frac{\ln 2}{k}$ $k = \frac{\ln 2}{11} = 6.3\%$	11 yr	\$17,539.32

4.5 # 27-47 odd

4.6 # 5, 7