Express in terms of sums and differences of simple logarithms, and simplify:

$$\log_a \sqrt{\frac{a^6b^8}{a^2b^5}} = \log_a \left(\frac{a^6b^8}{a^2b^5}\right)^{1/2} = \log_a \left(a^4b^3\right)^{1/2}$$

$$= \log_a \left(a^2b^{3/2}\right) = \log_a a^2 + \log_a b^{3/2} = 2 + \frac{3}{2}\log_a b$$

Express as a single logarithm, and simplify:

$$\ln x - 3[\ln(x-5) + \ln(x+5)]$$

$$\ln x - 3\ln[(x-5)(x+5)] = \ln x - 3\ln(x^2-25) =$$

$$= \ln x - \ln(x^2-25)^3 = \ln\left(\frac{x}{(x^2-25)^3}\right)$$

Exponential Decay

$$P(t) = P_0 e^{-kt}, \quad k > 0$$

P_0 = amount of substance at time $t = 0$

$P(t)$ = amount left at time t

k = decay rate

10. Carbon dating $P(t)=P_0e^{kt}$

statue lost 35% of carbon-14
how old is it?

Note: half-life of carbon-14 is 5750 yr.

$\frac{1}{2}P_0=P_0e^{k\cdot 5750}$

$\ln(\frac{1}{2})=k\cdot 5750$

$k=\frac{\ln\frac{1}{2}}{5750}=-0.00012$

$P(t)=P_0e^{-0.00012t}$

$P(t)=0.65P_0$

$0.65P_0=P_0e^{-0.00012t}$

$\ln 0.65=-0.00012t$

$t=\frac{\ln(0.65)}{-0.00012}=3590\text{yr}$

Limited Population Growth

Logistic Function

$$P(t)=\frac{a}{1+be^{-kt}}$$

$e^{-kt}=\frac{1}{e^{kt}}$

as $t\to\infty$, the population approaches a, so we have a horizontal asymptote $y=a$

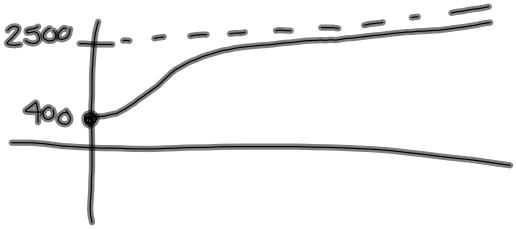
16. limited population growth in a lake

$P_0 = 400$ fish

limiting value is 2500

$$P(t) = \frac{2500}{1+5.25e^{-0.32t}}, \quad t \text{ in months}$$

a) Graph the function



b) Find the population after 0, 1, 5, 10, 15, & 20 months

$$P(10) = \frac{2500}{1+5.25 \cdot e^{-0.32(10)}} = \boxed{205 \text{ fish}}$$

20. When was the murder committed?

@ 12 pm, temp is 94.6°

(1hr later) @ 1 pm, temp is 93.4°

room temp is 70°

$P(t) = P_0 e^{kt}$

t = time in hours

P(t) = temperature above room temp

$93.4 - 70 = (94.6 - 70)e^{k \cdot 1}$

$23.4 = 24.6e^{k}$

$\ln\left(\frac{23.4}{24.6}\right) = k = -0.05$

$94.6 - 70 = (98.6 - 70)e^{-0.05t}$

$24.6 = 28.6e^{-0.05t}$

$\frac{24.6}{28.6} = e^{-0.05t}$

$\ln\left(\frac{24.6}{28.6}\right) = -0.05t$

$t = \frac{\ln\left(\frac{24.6}{28.6}\right)}{-0.05}$

$= 3h$

$\boxed{9 \text{ am}}$

4.6 #9, 15, 17

& old test #3