Express in terms of sums and differences of simple logarithms, and simplify:

$$\log_a\sqrt{\frac{a^6b^8}{a^2b^5}} = \log_a\left(\frac{a^6b^8}{a^2b^5}\right)^{1/2} = \log_a\left(a^4b^3\right)^{1/2}$$

$$= \log_a\left(a^2 b^{3/2}\right) = \log_a a^2 + \log_a b^{3/2} = \boxed{2+\frac{3}{2}\log_a b}$$

Express as a single logarithm, and simplify:

$$\ln x - 3[\underbrace{\ln(x-5)+\ln(x+5)}]$$

$$= \ln x - 3\ln[(x-5)(x+5)] = \ln x - 3\ln\left(x^2-25\right) =$$

$$= \ln x - \ln\left(x^2-25\right)^3 = \boxed{\ln\left(\frac{x}{(x^2-25)^3}\right)}$$

4.6

5. $k = 0.013$

$P_0 = 6{,}030{,}000$

$P(t) = 24{,}313{,}062{,}400$

$$\ln\left(e^{0.013t}\right) = 0.013t$$

$$\frac{\ln\left(\frac{24313062400}{6030000}\right)}{0.013} = \frac{\cancel{6030000}\,\cancel{e^{0.013t}}}{\cancel{6030000}} = x \qquad \cancel{0.013}$$

4.5

19. $3^x = 2^{x-1}$

$\ln 3^{(x)} = \ln 2^{(x-1)}$

$x \ln 3 = (x-1) \ln 2$

$x \ln 3 = x \ln 2 - \ln 2$

$x(\ln 3 - \ln 2) = -\ln 2$

$x = \dfrac{-\ln 2}{\ln 3 - \ln 2} \approx -1.710$

Exponential Decay

$P(t) = P_0 e^{-kt}$, $k > 0$

P_0 = amount of substance at time $t = 0$

$P(t)$ = amount left at time t

k = decay rate

10. Carbon dating

statue has lost 35% of its carbon-14
how old is it?

$P(t)=P_0e^{kt}$

*the half-life of carbon-14 is 5750 years.

$\frac{1}{2}P_0=P_0e^{kt}$

$\frac{1}{2}\cancel{P_0}=\cancel{P_0}e^{k\cdot 5750}$

$\ln\frac{1}{2}=k\cdot 5750$

$\text{decay rate}=\frac{\ln\frac{1}{2}}{\text{half-life}}$

$k=\frac{\ln\frac{1}{2}}{5750}\approx -0.00012$

$P(t)=0.65P_0$

$0.65\cancel{P_0}=\cancel{P_0}e^{-0.00012t}$

$\ln 0.65=-0.00012t$

$t=\frac{\ln 0.65}{-0.00012}\approx\boxed{3590\text{ years}}$

Limited Population Growth

Logistic Function

$$P(t)=\frac{a}{1+be^{-kt}}$$

$e^{-kt}=\frac{1}{e^{kt}}$

as $t\to\infty$, $e^{kt}\to\infty$

$\frac{1}{e^{kt}}\to 0$ & $P(t)\to a$

$\Rightarrow$ horizontal asymptote

$y=a$

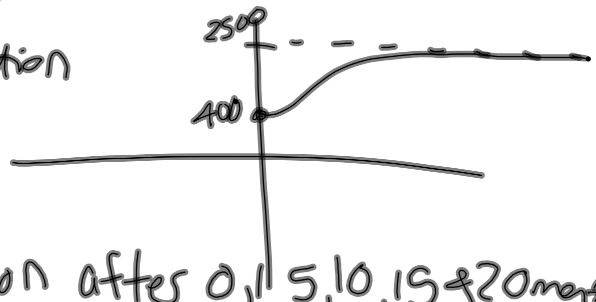
16. limited population growth in a lake

$P_0 = 400$ fish

limiting value is 2500

$$P(t) = \frac{2500}{1+5.25e^{-0.32t}} \quad , t \text{ in months}$$

a) Graph the function



b) Find the population after 0, 1, 5, 10, 15, & 20 months

$$P(15) = \frac{2500}{1+5.25e^{-0.32(15)}} \approx \boxed{2396 \text{ fish}}$$

20. When was the murder committed?

@ 12 pm temp is 94.6°

@ 1 pm (1 h later) temp is 93.4°

room temp is 70°

$P(t) = P_0 e^{kt}$

$(93.4° - 70°) = (94.6° - 70°)e^{k \cdot 1}$

$P(t)$ = temperature above room temp

$23.4 = 24.6e^{k}$

$\ln \frac{23.4}{24.6} = \ln e^{k}$

$94.6 - 70 = (98.6 - 70)e^{-0.05t}$

$k = \ln \frac{23.4}{24.6} = -0.05$

$24.6 = 28.6e^{-0.05t}$

$\boxed{9\text{am}}$

$$\frac{\ln \frac{24.6}{28.6}}{-0.05} = t = 3$$

4.6 #9, 15, 17

& old test #3