

b 1. $\log_a(MN) =$

- a. $\log_a M \log_a N$ b. $\log_a M + \log_a N$ c. $\log_a(M + N)$ d. $N \log_a M$

c 2. $\log_a 1 =$

- a. a b. 1 c. 0 d. e^a

d 3. $\log_a\left(\frac{M}{N}\right) =$

- a. $\frac{\log_a M}{\log_a N}$ b. $\frac{\ln M}{\ln a}$ c. $\log_a(M - N)$ d. $\log_a M - \log_a N$

b 4. $\log_a a =$

- a. a b. 1 c. 0 d. e^a

c 5. $\log x =$

- a. $\log_e x$ b. x c. $\log_{10} x$ d. $\log_1 x$

a 6. $\ln x =$

- a. $\log_e x$ b. x c. $\log_{10} x$ d. $\log_1 x$

b 7. $\log_a M^p =$

- a. $(\log_a M)^p$ b. $p \log_a M$ c. $(\log a)(M^p)$ d. $MP \log a$

b 8. $\log_a b =$

- a. $\frac{\log_b x}{\log_a x}$ b. $\frac{\log b}{\log a}$ c. $b \log a$ d. $\frac{\log a}{\log b}$

$$\log_a b = \frac{\log_c b}{\log_c a}$$

Prove that the function $f(x) = 3(x - 5)^2 + 7$ is not one-to-one.

$$f(6) = 10$$

$$f(4) = 10$$

1. Determine whether the function is one-to-one, and if it is one-to-one, find a formula for its inverse.

a. $f(x) = 2x - 1$

b. $f(x) = \frac{4}{x+7}$

$$f(a) = f(b)$$

$$\frac{4}{a+7} = \frac{4}{b+7}$$

$$4(b+7) = 4(a+7)$$

$$b+7 = a+7$$

$$b = a$$

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$$y = \frac{4}{x+7}$$

$$x = \frac{4}{y+7}$$

$$x(y+7) = 4$$

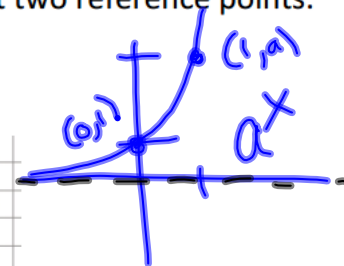
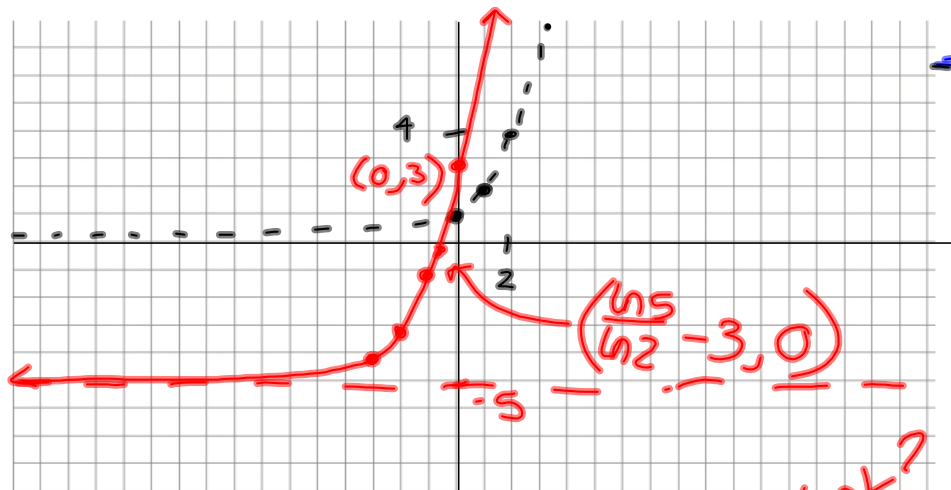
$$y+7 = \frac{4}{x}$$

$$y = \frac{4}{x} - 7$$

$$f^{-1}(x) = \frac{4}{x} - 7$$

2. Graph the function. Include labels for asymptotes and at least two reference points.

a. $f(x) = 2^{x+3} - 5$ 2^x , left 3, down 5



X-intercept?

$$0 = 2^{x+3} - 5$$

$$2^{x+3} = 5$$

Y-intercept?

$$2^{0+3} - 5$$

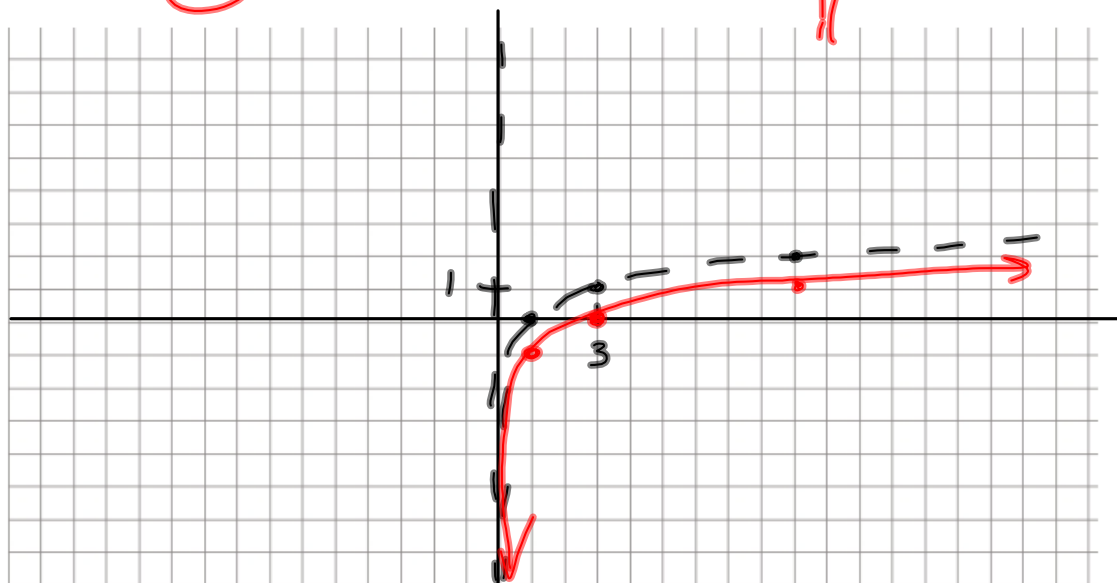
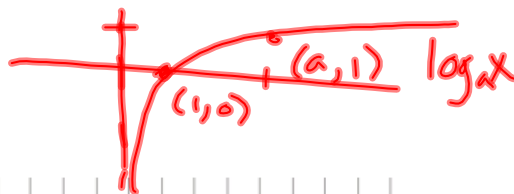
$$8 - 5 = 3$$

$$(x+3)\ln 2 = \ln 5$$

$$x+3 = \frac{\ln 5}{\ln 2}$$

$$x = \frac{\ln 5}{\ln 2} - 3$$

b. $f(x) = \log_3 x - 1$ down 1



3. Find each of the following without using a calculator. Show the intermediate steps that led to the answer. Give exact answers.

a. $\log_2 \frac{1}{4} = \boxed{-2}$ $2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

b. $\log \sqrt{10} = \log_{10} 10^{1/2} = \boxed{\frac{1}{2}}$

c. $\ln \frac{1}{e^5} = \log_e e^{-5} = \boxed{-5}$

4. Find the logarithm using natural logs and the change-of-base formula. Give an exact answer in terms of logs and ~~an approximate answer to four decimal places~~.

$$a. \log_3 12 = \frac{\ln 12}{\ln 3} \approx$$

$$b. \log_{100} 15 = \frac{\ln 15}{\ln 100} \approx$$

5. Express as a single logarithm.

$$a. \frac{2}{3} [\ln(x^2 - 9) - \ln(x + 3)] + \ln(x + y)$$

$$= \frac{2}{3} \ln \frac{x^2 - 9}{x + 3} + \ln(x + y)$$

$$= \frac{2}{3} \ln \frac{(x-3)\cancel{(x+3)}}{\cancel{x+3}} + \ln(x + y)$$

$$= \frac{2}{3} \ln(x-3) + \ln(x + y) = \ln(x-3)^{2/3} + \ln(x + y)$$

$$= \ln \left[(x-3)^{2/3} (x + y) \right]$$

$$b. \ln 2x + 3(\ln x - \ln y)$$

$$\ln \left[2x \left(\frac{x}{y} \right)^3 \right] = \ln \left(\frac{2x^4}{y^3} \right)$$

6. Express in terms of sums and differences of logs.

a. $\log \sqrt{x^3 y}$

$$\begin{aligned} \text{b. } \log_c \sqrt[3]{\frac{y^3 z^2}{x^4}} &= \log_c \left(\frac{y^3 z^2}{x^4} \right)^{1/3} = \log_c \frac{y z^{2/3}}{x^{4/3}} \\ &= \log_c y + \log_c z^{2/3} - \log_c x^{4/3} \\ &= \boxed{\log_c y + \frac{2}{3} \log_c z - \frac{4}{3} \log_c x} \end{aligned}$$

7. Given that $\log_b 2 \approx 0.693$, $\log_b 3 \approx 1.099$, and $\log_b 5 \approx 1.609$, find the following to the nearest thousandth.

$$\begin{aligned} \text{a. } \log_b \frac{1}{6} &= \log_b 6^{-1} = -\log_b 6 = -\boxed{\log_b (2 \cdot 3)} \\ &= -(\log_b 2 + \log_b 3) = \boxed{-\log_b 2 - \log_b 3} \end{aligned}$$

$$\begin{aligned} \text{b. } \log_b 30 &= \log_b (2 \cdot 3 \cdot 5) \\ &= \log_b 2 + \log_b 3 + \log_b 5 \end{aligned}$$

$$\begin{aligned} \log_b \frac{1}{2 \cdot 3} &= \\ \log_b 1 - \log_b 2 - \log_b 3 & \end{aligned}$$

8. Simplify.

$$\text{a. } 5^{\log_5(4x-3)} = 4x-3$$

$$\text{b. } \log_b \sqrt{b^3} = \log_b (b^3)^{1/2} = \log_b b^{3/2} = \boxed{\frac{3}{2}}$$

9. Solve for x. Find an exact answer algebraically.

a. $\log_2(x + 1) + \log_2(x - 1) = 3$

$\log_2[(x+1)(x-1)] = 3$

$\log_2(x^2 - 1) = 3$

$2^3 = x^2 - 1$

$9 = x^2$

$\pm 3 = x$

$x = 3$

$\log_a b = c$
 \iff
 $a^c = b$

b. $5^{4x-7} = 125$

$5^{4x-7} = 5^3 \implies 4x - 7 = 3$

$4x = 10$

$x = \frac{10}{4} = \frac{5}{2}$

10. In 1984, the average cell phone price was \$3395, and in 2002, it was \$145. Assuming the average price of a cell phone decreased according to the exponential model,

a. Find the value of k, and write an exponential function that describes the average price of a cell phone after time t, in years, where t is the number of years since 1984.

$P_0 = 3395$ $145 = 3395e^{-k \cdot 18}$ $P(t) = P_0 e^{-kt}$

$P(18) = 145$ $\frac{145}{3395} = e^{-18k}$

$\ln\left(\frac{145}{3395}\right) = -18k$

$k = -\frac{\ln\left(\frac{145}{3395}\right)}{18}$

$P(t) = 3395 e^{\frac{\ln\left(\frac{145}{3395}\right)}{18} t}$

b. Estimate the price of a cell phone in 2006 and 2008 to the nearest dollar.

$P(29) = 3395 e^{\frac{\ln\left(\frac{145}{3395}\right)}{18} \cdot 29}$

terms of e & ln

$\approx \$21$

c. At this decay rate, in what year will the price be \$39?

$39 = 3395 e^{\frac{\ln\left(\frac{145}{3395}\right)}{18} \cdot t}$

$\frac{39}{3395} = e^{\frac{\ln\left(\frac{145}{3395}\right)}{18} \cdot t}$

$\ln\left(\frac{39}{3395}\right) = \frac{\ln\left(\frac{145}{3395}\right)}{18} t$

$t = \frac{\ln\left(\frac{39}{3395}\right)}{\left(\frac{\ln\left(\frac{145}{3395}\right)}{18}\right)}$

year is +1984.