b 1. $\log_a(MN) =$

a. $\log_a M \log_a N$ b. $\log_a M + \log_a N$ c. $\log_a(M+N)$ d. $N \log_a M$

c 2. $\log_a 1 =$

a. a b. 1 c. 0 d. e^a

d 3. $\log_a\left(\frac{M}{N}\right) =$

a. $\frac{\log_a M}{\log_a N}$ b. $\frac{\ln M}{\ln a}$ c. $\log_a(M-N)$ d. $\log_a M - \log_a N$

b 4. $\log_a a =$

a. a b. 1 c. 0 d. e^a

c 5. $\log x =$

a. $\log_e x$ b. x c. $\log_{10} x$ d. $\log_1 x$

a 6. $\ln x =$

a. $\log_e x$ b. x c. $\log_{10} x$ d. $\log_1 x$

b 7. $\log_a M^P =$

a. $(\log_a M)^P$ b. $P \log_a M$ c. $(\log a)(M^P)$ d. $MP \log a$

b 8. $\log_a b =$

a. $\frac{\log_b x}{\log_a x}$ b. $\frac{\log b}{\log a}$ c. $b \log a$ d. $\frac{\log a}{\log b}$

$\log_a b = \frac{\log_c b}{\log_c a}$

Prove that the function $f(x) = 3(x-5)^2 + 7$ is not one-to-one.

$f(6) = 10$

$f(4) = 10$

1. Determine whether the function is one-to-one, and if it is one-to-one, find a formula for its inverse.

a. $f(x) = 2x - 1$

$f(a) = f(b)$

$\Rightarrow a = b$

b. $f(x) = \frac{4}{x+7}$

$\frac{4}{a+7} = \frac{4}{b+7}$

$4(a+7) = 4(b+7)$

$a+7 = b+7$

$a = b$

$\therefore f$ is one-to-one

$y = \frac{4}{x+7}$

$x = \frac{4}{y+7}$

$x(y+7) = 4$

$y+7 = \frac{4}{x}$

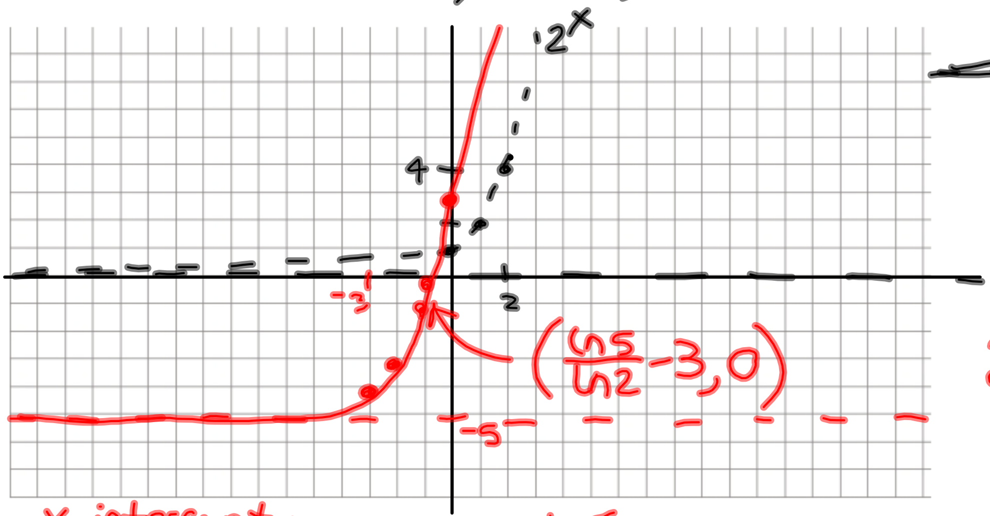
$y = \frac{4}{x} - 7$

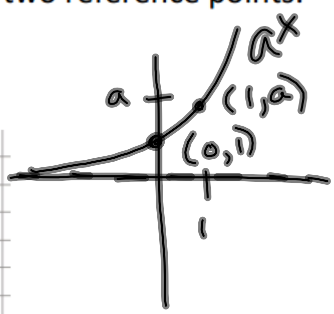
$\boxed{f^{-1}(x) = \frac{4}{x} - 7}$

2. Graph the function. Include labels for asymptotes and at least two reference points.

a. $f(x) = 2^{x+3} - 5$

2^x, left 3, down 5





y-intercept:

$2^{0+3} - 5$

$2^3 - 5$

$8 - 5 = 3$

x-intercept:

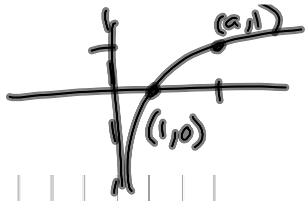
$0 = 2^{x+3} - 5$

$5 = 2^{x+3}$

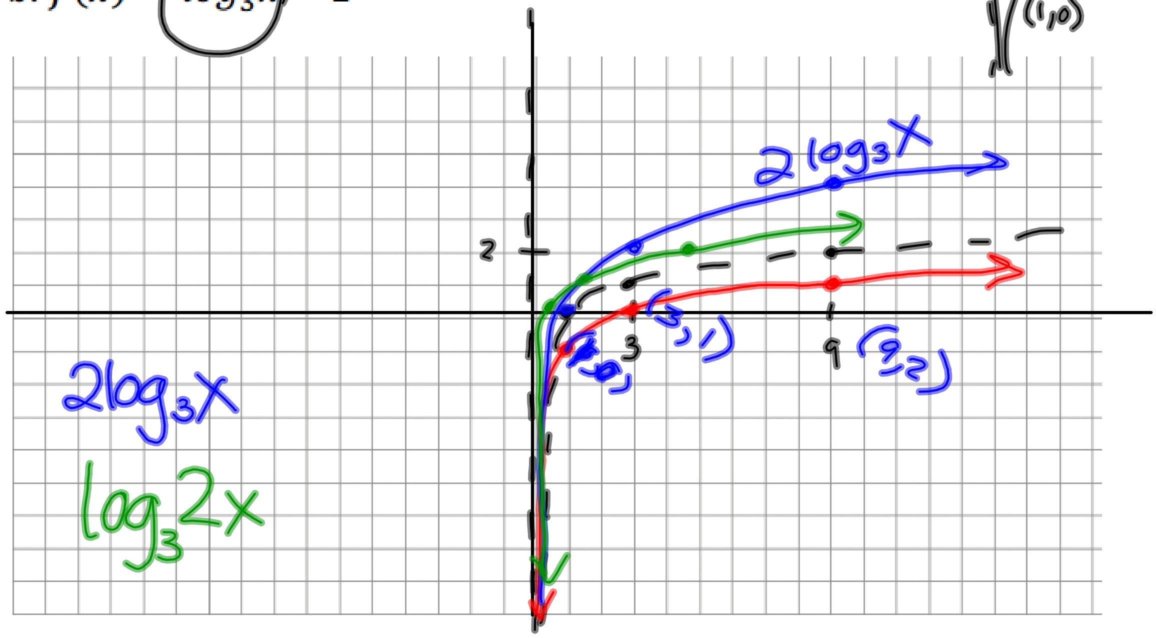
$\ln 5 = (x+3)\ln 2$

$x+3 = \frac{\ln 5}{\ln 2}$

$x = \frac{\ln 5}{\ln 2} - 3$



b. $f(x) = \log_3 x - 1$ down 1



3. Find each of the following without using a calculator. Show the intermediate steps that led to the answer. Give exact answers.

a. $\log_2 \frac{1}{4} = \boxed{-2}$ $\quad 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

b. $\log \sqrt{10} = \log_{10} 10^{1/2} = \boxed{\frac{1}{2}}$

c. $\ln \frac{1}{e^5} = \log_e e^{-5} = \boxed{-5}$

4. Find the logarithm using natural logs and the change-of-base formula. Give an exact answer in terms of logs ~~and an approximate answer to four decimal places.~~

a. $log_3 12 = \frac{\ln 12}{\ln 3}$

b. $log_{100} 15 = \frac{\ln 15}{\ln 100}$

5. Express as a single logarithm.

a. $\frac{2}{3}[\ln(x^2-9) - \ln(x+3)] + \ln(x+y)$

$= \frac{2}{3} \ln \frac{x^2-9}{x+3} + \ln(x+y) = \frac{2}{3} \ln \frac{\cancel{(x+3)}(x-3)}{\cancel{x+3}} + \ln(x+y)$

$= \frac{2}{3} \ln(x-3) + \ln(x+y) = \ln(x-3)^{2/3} + \ln(x+y) =$

$= \ln\left[(x-3)^{2/3}(x+y)\right]$

b. $\ln 2x + 3(\ln x - \ln y)$

6. Express in terms of sums and differences of logs.

a. $\log\sqrt{x^3y}$

b. $\log_c\sqrt[3]{\frac{y^3z^2}{x^4}} = \log_c\left(\frac{y^3z^2}{x^4}\right)^{1/3} = \log_c\frac{yz^{2/3}}{x^{4/3}}$

$= \log_c y + \log_c z^{2/3} - \log_c x^{4/3}$

$\boxed{= \log_c y + \frac{2}{3}\log_c z - \frac{4}{3}\log_c x}$

7.Given that $\log_b 2$ ~~≈ 0.693~~, $\log_b 3$ ~~≈ 1.099~~, *and* $\log_b 5$ ~~≈ 1.609~~, find the following ~~to the nearest thousandth.~~

a. $\log_b\frac{1}{6} = \log_b 6^{-1} = -\log_b 6 = -(\log_b(2\cdot3))$

$= -(\log_b 2 + \log_b 3) = \boxed{-\log_b 2 - \log_b 3}$

$\log_b\frac{1}{2\cdot3} = \log_b 1 - (\log_b 2 + \log_b 3)$

$\log_b 1 = 0$

b. $\log_b 30 = \log_b(2\cdot3\cdot5)$

$\boxed{= \log_b 2 + \log_b 3 + \log_b 5}$

8. Simplify.

a. $5^{\log_5(4x-3)} = 4x-3$

b. $\log_b\sqrt{b^3} = \log_b(b^3)^{1/2} = \log_b b^{3/2} = \boxed{\frac{3}{2}}$

9. Solve for x. Find an exact answer algebraically.

a. $\log_2(x+1) + \log_2(x-1) = 3$

$\log_2[(x+1)(x-1)] = 3$

$\log_2(x^2-1) = 3$

$2^3 = x^2 - 1$

$9 = x^2$

$x = \pm 3$

$\boxed{x = 3}$

$\boxed{\log_a b = c \iff a^c = b}$

b. $5^{4x-7} = 125$

$5^{4x-7} = 5^3$

$4x - 7 = 3$

$4x = 10$

$x = 10/4 = \boxed{5/2}$

10. In 1984, the average cell phone price was $3395, and in 2002, it was $145. Assuming the average price of a cell phone decreased according to the exponential model, $P(t) = P_0 e^{kt}$

a. Find the value of k, and write an exponential function that describes the average price of a cell phone after time t, in years, where t is the number of years since 1984.

$P_0 = 3395$

$P(18) = 145$

$145 = 3395e^{k \cdot 18}$

$\frac{145}{3395} = e^{k \cdot 18}$

$\ln\frac{145}{3395} = k \cdot 18$

$k = \frac{\ln\frac{145}{3395}}{18}$

$\boxed{P(t) = 3395e^{\frac{1}{18}\ln\frac{145}{3395}t}}$

b. Estimate the price of a cell phone in ~~2006 and 2008 to the nearest dollar.~~ 2013

$P(29) = \boxed{3395e^{(\frac{1}{18}\ln\frac{145}{3395}) \cdot 29}}$

$\approx \$21.11$

c. At this decay rate, in what year will the price be $39?

$39 = 3395e^{(\frac{1}{18}\ln\frac{145}{3395})t}$

$\frac{39}{3395} = e^{(\frac{1}{18}\ln\frac{145}{3395})t}$

$\ln\frac{39}{3395} = (\frac{1}{18}\ln\frac{145}{3395})t$

$t = \frac{\ln\frac{39}{3395}}{\frac{1}{18}\ln\frac{145}{3395}} = \boxed{\frac{18(\ln 39 - \ln 3395)}{\ln 145 - \ln 3395}}$

2009!